1 Demand analysis

- When the world price of oil increases, should the government reduce gas taxes to keep the retail price of gas the same or should the government give a lump sum tax credit to consumers?

- Individual demand:

  1. Income effects
  2. Substitution effects
  3. Total effect of price change

- Market demand:

  1. Derive from individual demand
  2. Price elasticity and changes in expenditure
  3. Consumer surplus

2 Consumer's problem:

$$Max\, U(F, C)$$

subject to:

$$P_C C + P_F F \leq I$$

3 Income effect

- Normal versus inferior good
- Engel curve

4 Substitution effect of price change

- Minimize cost when price changes subject to utility held constant.

- What is the change in quantity when relative prices change?

- Why do you care about substitution effect? Comparing cost of living.

- Buy more of good that is relatively cheaper.

- Buy less of good that is relatively more expensive.

5 Total quantity change due to price change

- Total price change decomposes into (1) Income and (2) Substitution effect.

- Assume price of C increases.

  1. Income effect can be negative or positive.

  2. Substitution effect is negative.

- Effect of subsidizing electricity prices versus tax rebate for consumers when cost of producing electricity rises.

6 Market demand

- Market demand is horizontal sum of individual demand.

- Demand elasticity and expenditure:

$$
\begin{aligned}
E &= PQ \\
\Delta_E &\simeq P\Delta_Q + Q\Delta_P \\
\frac{\Delta_E}{\Delta_P} &= \frac{P\Delta_Q}{\Delta_P} + Q \\
\frac{\Delta_E}{\Delta_P} &= Q(\frac{P\Delta_Q}{Q\Delta_P} + 1) \\
\frac{\Delta_E}{\Delta_P} &= Q(\varepsilon_P + 1)
\end{aligned}
$$

- ε_P is inelastic, $\frac{\Delta_E}{\Delta_P}$ is positive.

- ε_P is unitary, $\frac{\Delta_E}{\Delta_P}$ is zero. ε_P is elastic, $\frac{\Delta_E}{\Delta_P}$ is negative.

7 Consumer surplus

- We want a measure of change in consumer welfare as price of commodity changes.

- Consumer surplus measures the difference between the maximum amount that a consumer is willing to pay for a good and the what the consumer actually pays.

- For a first unit, the consumer may be willing to pay \$20 but only has to pay \$14. So his consumer surplus from the first unit is \$6. After he buys the first unit, he is willing to pay \$19 for a second unit but he only still has to pay \$14. So his consumer surplus from the second unit is \$5. Total consumer surplusf for two units is \$9.

- The last unit he buys gives him a consumer surplus of approximately zero.

- Consumer surplus is the area under the individual ordinary demand curve minus his expenditure.

- Does the price of the good reflect its consumer surplus?

- Does the fall in consumer surplus measure how much you have to compensate the consumer for an increase in price?

- The change in welfare approximation is exact if there is no income effect from the price change or if the income effect is small. E.g. this is good for measuring changes in prices where expenditure on the good is a small fraction of income. So change in CS is good measure of welfare changes for changes in bread prices and less good for changes in housing prices. Good measure for preserving the forests for tourists (like us) but not good for hunters or loggers.

- You can figure out the direction of the bias if you know the direction of the income effect. But most applied analysts use CS as a first approximation.

8 Cobb Douglas example

$$U(F, C) = aF^{\alpha}C^{\beta}$$

Consumer solves

$$\max_{F,C} U(F, C) = aF^{\alpha}C^{\beta}$$

subject to

$$P_F F + P_C C \leq I$$

$$U_C = \beta a F^{\alpha} C^{(\beta-1)}$$
$$U_F = \alpha a F^{(\alpha-1)} C^{\beta}$$

We also know that at the optimum:

$$\frac{U_C^*}{U_F^*} = \frac{P_C}{P_F}$$
$$\frac{\beta a F^{*\alpha} C^{*(\beta-1)}}{\alpha a F^{*(\alpha-1)} C^{*\beta}} = \frac{\beta F^*}{\alpha C^*} = \frac{P_C}{P_F}$$
$$\frac{\beta(\frac{I - P_C C^*}{P_F})}{\alpha C^*} = \frac{P_C}{P_F}$$
$$C^* = \frac{\beta}{\alpha + \beta} \frac{I}{P_C}$$
$$F^* = \frac{\alpha}{\alpha + \beta} \frac{I}{P_F}$$

C^* is increasing in I and decreasing in P_C. It does not depend on P_F which is peculiar to the Cobb Douglas utility function.

C^* does not depend on a. What does this mean?

To solve consumer's optimization problem,

substitute budget constraint into utility function and consumer solves:

$$\max_C U(\frac{I - P_C C}{P_F}, C) = \ a(\frac{I - P_C C}{P_F})^{\alpha} C^{\beta}$$

$$\begin{aligned}
\frac{\partial U}{\partial C} &= \frac{\partial\ a(\frac{I - P_C C}{P_F})^{\alpha} C^{\beta}}{\partial C} \\
&= \alpha a(\frac{I - P_C C}{P_F})^{\alpha - 1} C^{\beta}(\frac{-P_C}{P_F}) + \beta a(\frac{I - P_C C}{P_F})^{\alpha} C^{\beta - 1}
\end{aligned}$$

At the maximum:

$$\begin{aligned}
\frac{\partial U}{\partial C}|_{C^*} &= 0 \\
\alpha a(\frac{I - P_C C}{P_F})^{\alpha - 1} C^{\beta}(\frac{-P_C}{P_F}) + \beta a(\frac{I - P_C C}{P_F})^{\alpha} C^{\beta - 1} &= 0 \\
\frac{\beta(I - P_C C^*)}{\alpha C^*} &= P_C
\end{aligned}$$

Marginal utility of income

$$V(P_F, P_C, I) = U(F^*, C^*) = a(\frac{\beta}{\alpha+\beta}\frac{I}{P_F})^{\alpha}(\frac{\alpha}{\alpha+\beta}\frac{I}{P_C})^{\beta}$$
$$= a(\frac{\beta}{\alpha+\beta})^{\alpha}(\frac{\alpha}{\alpha+\beta})^{\beta}\frac{I^{\alpha+\beta}}{P_C^{\alpha}P_F^{\beta}}$$

Utility is increasing in income and decreasing in prices.

$$V_I = (\alpha+\beta)a(\frac{\beta}{\alpha+\beta})^{\alpha}(\frac{\alpha}{\alpha+\beta})^{\beta}\frac{I^{\alpha+\beta-1}}{P_C^{\alpha}P_F^{\beta}} > 0$$