The Market for Intellectual Property: Evidence from the Transfer of Patents

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Abstract

This paper quantifies what are the gains from trade and the costs of adopting technology in the market for U.S. patents. We estimate the parameters of a model of the transfer and renewal of patents using new data on the transfer of their ownership. The fundamental basis of our identification strategy is the interplay of the cost of technology transfer with the revenue and the age of a patent. The cost of technology transfer creates a selection effect so that better patents are more likely to be traded. This effect accounts for the discontinuous increase in the probability of an active patent being traded immediately after renewal and for why traded patents are more likely to be retraded and less likely to be allowed to expire. There is also a horizon effect that accounts for why the probability of active patents decreases as they get closer to their expiration date. A shorter horizon implies less time to amortize the costs of technology transfer.

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1 Introduction

Nowadays intangibles assets (i.e., patents, trademarks, copyrights, etc) have become an increasing large share of develop economies. For instance, in the early 1980s they represented 38 percent of the portfolios of U.S. firms. In the mid 1990s this share rose to 70 percent (WIPO [40])). However, we know little about measuring them, whether there are significant spillovers in their use, how and why their ownership is reallocated between firms and what are the gains from their reallocation. In this paper, we focus on the dissemination of patents across firm's boundaries and its benefits.

We seek especially to quantify the gains from trade as well as the costs of adopting technology in the market for patents. Patents are traded because some firms are more productive than others in their use. By gains from trade we mean the differential in the value of a patent between its current owners and the firm that actually acquires it (i.e., the net private benefits from the observed reallocation of the ownership of patents). Reallocating a patent is likely to involve a resource cost, a cost of adopting a technology. An investment to make an efficient use of the acquired knowledge (i.e., the reestructuring of the firm to adopt the new technology, expenditures in R&D, the hiring of engineers, etc.). At the aggregate level, these gains from trade and costs might carry important economic consequences in the make or buy decision of firms, the development of a market for technology and the long-run prospects of technological change.

The fundamental basis of our empirical strategy for quantifying the gains from trade and the cost of technology transfer is the interplay of this cost with the revenue and the age of a patent. First, we argue that costs of technology transfer create a selection effect so that better patents are more likely to be traded. This selection effect accounts for the reason that traded patents are more likely to be retraded and less likely to expire. In particular, when in the model the costs of technology transfer are nule, then there is no selection in the trading decision. The latter would imply that the probabilities of being traded of previously traded or not previously traded patents would be no different from each other. Second, we show that costs of adopting technology also involve a horizon effect that explains why the probability of active patents decreases as they get closer to their expiration date. A shorter horizon implies less time to amortize the costs. Now, notice that in the counterfactual case that the costs were nule or proportional to the present value of a patent, then there would be no horizon effect and the predicted probability of a patent being traded (i.e., transfer rate) would be independent of its age. However, the observed patterns in the data are very different than the above metioned implications of nule costs of technology transfer. Therefore, the estimate of the cost of technology transfer is pinned down from both the decreasing shape of the probability of active patents being traded and the difference between the probability of active patents being traded (allowed to expire) conditional on having being previously traded and not.

Another important element of the identification strategy are the discontinous jumps in the probability of an active patent being traded immediately after each renewal date. We argue that such discontinous jumps in the data are implied by the increased in the average quality of the stock of active patents immediately after a renewal decision as well as the effect that better patents are more likely to be traded. In particular, at each renewal event patents with the lowest revenue are let to expire by their owners, thus immediately after each renewal date the average revenue of existing patents is higher than the revenue immediately before a renewal date. Since patents with higher revenue are more likely to be traded, then the selection effect accounts for the discontinuous increase in the transfer rate after each renewal date.

In addition, the stochastic process that determines the improvement in the use of a patent by a buyer is identified from the level of the transfer rate, i.e., the higher the mean of the improvement factor, the more likely patents are traded and then the higher is the level of the transfer rate.

The parameters of the model are estimated using the simulated general method of moments (McFadden [27] and Pakes and Pollard [31]). We use data on the transfer and renewal of patents. While our research question is rather different than the questions addressed by the literature on patent value; -we estimate the gains from trade in the market for patents while previous scholars focused on estimating the value of patents-, our work shares important methodological aspects with them (Schankerman and Pakes [33], Pakes [29], Putnam [32] and Lanjouw [23]). In particular, the patent renewal data allows us to pin down the Dollar counterpart of our estimates.

The market for patents plays a significant role in the life cycle of a patent. We find that the market accounts for 14.2 percent of the estimated average value of a traded patent. There are large differences between the present value of a newly minted patent depending on whether is eventually traded or not. For example, the average value of a traded and non-traded patent is, respectively $100,041 and $29,660. Moreover, costs of technology transfer are important. We estimate this cost at $12,893, a 12.8 percent of the value of a traded patent. We also share empirical results with the previous literature. The distribution of patent value is quite skewed: While the average value of a patent is $US (2003) 37,767, the bottom 50 percent of all granted patents only account for 10.2 percent of the cumulative value of all patents and the bottom 90 percent accounts for 51.8 percent of the total value.

Our earlier work (Serrano, [34]) documents the patterns that characterize the transfer of patents and develops a dynamic model that highlights the consequences of costs of adopting technology in the trading of patents. The work is distinct to the previous literature in that it makes use of data on the transfer of patents. The transfer of patents is a significant event of the life cycle of a patent. Nearly 20 percent of all U.S. patents issued to small innovators (i.e., firms that were issued no more than 5 patents in a given year) are traded at least once over their life cycle. We found five patterns. First, the number of patents traded as a percentage of active patents (i.e., the transfer rate) varies over the life cycle. It monotonically decreases with the exceptions of the renewal dates. In particular, immediately after renewal, transfer rates discontinuously increase. Second, the number of expired patents at a renewal date as a percentage of all active patents (i.e., expiration rate) monotonicall increases as a function of age. Third, patents with higher number of total citations received by a given renewal date are less likely to be allowed to expire[1]. Fourth, patents with higher number of total citations received are more likely to be traded. Fifth, patents that have been previously traded, and in particular the recently traded, are more likely to be retraded and less likely to expire.

There is no previous work quantifying the benefits of a market for patents. There is, however, an important literature on markets for intellectual property. The literature can be summarized into four groups. One strand aims to demonstrate evidence of the existence of this market. The method generally used has been the analysis of industry case studies, such as the works collected in Arora, Fosfuri and Gambardella [6]. In addition, a sequence of papers by Lamoreaux and Sokoloff [25], [26] provide an account of organized markets for technology in the late 19^{th} and early 20^{th} century, prior to the growth of in house R&D laboratories by large firms. The second strand of the literature has suggested the existence of potential gains from specialization and diffusion of technology (Arrow [7], Arora, Fosfuri and Gambardella [6]). The third strand has analyzed the limitations of the market, such as the appropiation problems in the transfer of knowledge (Arrow [8], Anton and Yao [3] [4], Teece [37], Williamson [39]), Gans and Stern [11] and the cost of transfer of technology (Teece [36]). The four strand focuses on strategic considerations and the design and use of incentives in contracts of technology transfer. Katz and Shapiro [22], Gallini and Winter [12], and Shepard [35] consider the transfer of technology as a strategic decision; Aghion and Tirole [2] and Arora [5] study the design of licensing contracts in terms of incentives.

There is also an extensive empirical literature investigating patent data (BLS [10], Griliches [13], [14], Pakes and Griliches [30], and Hall, Griliches and Hausman [18], Jaffe,

[1]Each patent when granted lists references to previous patents, that is citations made. Instead, citations received by a patent is the number of times that this patent has been referenced by other patents. Previous empirical studies on patents have found that citations received by a patent is a measure of the economics value of a patent.

Henderson and Tratjenberg [19] and Tratjenberg, Henderson and Jaffe [38], Hall, Jaffe and Tratjenberg [16], Hall, Jaffe and Tratjenberg [17] and others). Our work is different in that it uses data on transfers of patents. The U.S. patent office registers transfers of patents in the same way that counties register the transfer of houses. As we show here, the market for trade in patents is large. In our study, we make use of all the records of titles transferred and link this information to the basic patent data (e.g., patent's grant date, renewals, citations received, etc.) that others have used.

In addition, our work opens new avenues of research. First, to study the sources of innovation and to characterize who are the buyers and sellers of technology. In particular, to trace the flow of technology transfer and to analyze whether small firms specialize in the creation of innovations that eventually are sold to their larger counterparts. Second, to examine to what extent a higher level of patent protection has facilitated specialization and, consequently trade in patents. Lastly, to evaluate the use of taxation on intellectual property transfer to promote innovation. These questions have not been previously empirically addressed due to a lack of data on how patents are traded.

This paper is organized as follows. Section 2 explains the data and presents the stylized facts. Section 3 lays out the model. Section 4 solves the model and links the data patterns with the results of the model. Section 5 presents the estimation strategy and discuss the identification of the estimates. Section 6 shows the estimation results and quantifies what are the gains from trade in the market for patents. Section 7 concludes the paper. Finally, all proofs, some extensions of the model and a data summary are included in the Appendix.

2 Data

A patent for an invention is the grant of a property right to the inventor in order to exclude others from making, using, or selling the invention. The life cycle of a patent begins at the grant date.[2] By the end of year 4, 8 and 12 upon the grant date renewal fees are due. If they are not paid, then the patent expires.[3] Such renewal events have been studied for patents granted in European countries in an extensive and important literature (Schankerman and Pakes [33], Pakes [29], Lanjouw [23] and others).

Another event that can happen in the life cycle of a patent is what the U.S. patent office calls "reassignments", and what we will call a "transfer" or "trade". In principle,

[2]The term of new patents applied for prior to 1995 was 17 years from their grant date. This term was subsequently modified to 20 years from the date in which the patent application was filed.

[3]Renewal fees are due by the end of years 4, 8 and 12 since the grant date of the patent. The USPTO began charging renewal fees in 1984 on patents applied for after December 12, 1980.

the event can happen many times during the life of a single patent. The U.S. patent office maintains a registry of these events. We have obtained these records for all transfers that occurred from 1981 to 2002, of which there were 1,041,083. The records have information about patent numbers, making it possible to merge the patent level data on renewals and citations that has been used in the previous literature. The details of the procedures we used to deal with the transfer data are explained in Serrano [34].

A particular issue we treat in detail in Serrano [34] is that some of the transfers recorded with the patent office are administrative events, like a name change, as opposed to a true economic transfer between two distinct parties. Fortunately, for each transaction there is a data field that records the "brief", which is the nature of the trade. We separate out traded patents where the reason is a name change, a security interest, a correction, etc. The remaining accounts for 508,756 patents, 49% of all traded patents.

A second issue is that in cases where there is a merger between two large companies, patents are traded in large blocks. When Burroughs Corporation merged with Sperry Corporation to create Unisys Corporation in September 1986, this event appears in my data as transactions totalling 2261 patents (the largest single transaction includes 1702 patents). Our theoretical analysis will focus on decision making at the *patent* level. There are costs and benefits of transferring a particular patent. Obviously, in a wholesale trade such as Burroughs merging with Sperry, the decision making is not at the level of a single patent. To parallel our focus in the theory, in our empirical analysis we focus on *small innovators*. In doing so, the economic forces that we highlight will be more salient than in transactions involving the likes of Burroughs or Sperry.[4] In addition, small innovators are interesting in their own right, given the importance they play in the innovation process (Arrow [9], Acs and Audretsch [1]). Indeed, we operationalize this focus on small innovators by restricting attention to patents granted to firms with no more than 5 patents granted to them that year.

The dataset we have compiled is a panel of patents detailing their histories of trade and renewal decisions. The panel contains patents that were applied for after December 12, 1980 and issued since January 1, 1983 to U.S. or foreign businesses.[5] In addition, it has characteristics such as citations received, industry to which the patented technology belongs, size

[4] A companion paper, "Measuring the Transfer of Patents" shows that patents granted to large corporations are more likely to be traded for other reasons than the technology that they represent. For instance, they can be recorded as a result of large acquisitions pursued to increase the buyer's market share in a particular product, etc.

[5] Patents applied for after December 12, 1980 are subject to renewal fees. To create a comphensive sample we consider January 1, 1983 as the starting grant date of the patents contained in the panel. Finally, issued to U.S. or foreign business means that at the date the patent was granted, the owner was a U.S. or foreign business.

of the firm that was the owner at the grant date, and other relevant information.

The panel includes 453,683 patents granted to small innovators. This sample contains about a third of all granted patents to U.S. or foreign businesses.

The next section presents the key patterns of the transfer of patents.

2.1 Patterns

This section presents the basic patterns that describe the underlying quality of traded patents, how the transfer rate varies over the life cycle of a patent, and the effects of a transfer on the renewal and trading decision.[6] The key patterns are the following:

1. The transfer rate[7] monotonically decreases following the first year after a patent has been granted, with the exceptions of the renewal dates. Immediately after renewal, the transfer rate discontinuously increases. Moreover, the transfer rate increases during the application period of a patent.
2. The expiration rate[8] monotonically increases as a function of the renewal dates.
3. Patents with higher number of total citations received by a given renewal date are less likely to be allowed to expire.
4. Patents with higher number of total citations received by a given age are more likely to be traded.
5. Among patents of the same age, those that have been previously traded, in particular those recently traded, are more likely to be retraded and less likely to expire.

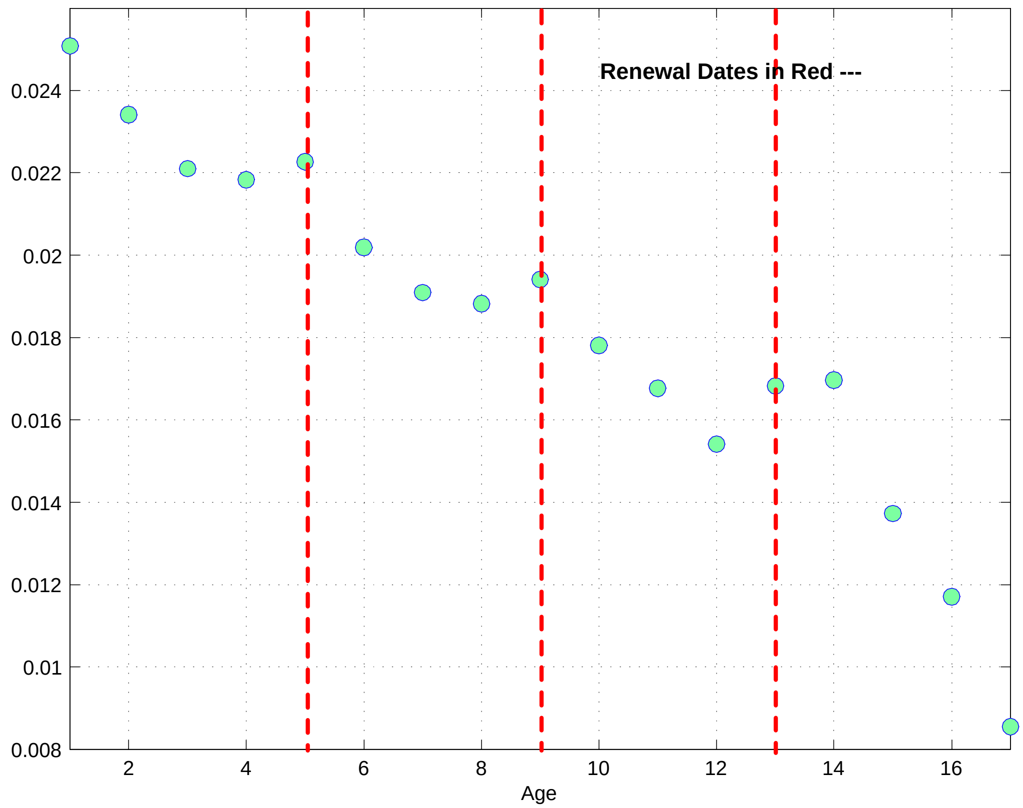
Pattern #1: The transfer rate monotonically decreases from the grant date until the end of a patent's life except at the renewal dates. Immediately after renewal, this rate increases abruptly. For instance, the proportion of traded patents drops from 2.51 to 2.18% respectively, from age 1 to age 4. Then in age 5 it increases to 2.23 and in age 6 drops again to 2.02%. This evidence is consistent for all three renewal dates in which renewal fees are due according to the U.S. patent system, and controlling by patent classes.

[6]For explanation, the age of a patent is defined as follows. Its age when it is traded is the number of years between the trade date and the grant date. In particular, if a patent was traded during its second year of life (e.g., 17 or 22 months since being issued), I consider that the patent was traded at age 2.

[7]Transfer rate at a given patent age is defined as the proportion of patents that are traded conditional on having survived up to that period.

[8]Expiration rate at a given patent age is defined as the proportion of patents that are expired conditional on having survived up to that period.

Figure 1: The Proportion of Traded Patents Conditional on Renewal



Pattern #2: The expiration rate monotonically increases as a function of the renewal dates. In Table 1 we show the expiration rate at each renewal date. For instance, among patents of age 5, 18.08% are allowed to expire by that date. Four years after, at age 9, 28.39% of the remaining patents are allowed to expire. The last renewal fee, due by the beginning of age 13, implies that 32.02% of all active patents become not active. The finite horizon of a patent life together with the "depreciation" of their per period revenue due to imatation of the arrival of superior technologies might the forces that are behind the increasing pattern of the expiration rate.

Pattern #3: Patents with higher number of total citations received by a given age are less likely to be allowed to expire. In Table 1A, we present the predicted probabilities of calculated with the estimates of a logit model. We have regressed the renewal decision on renewal date dummies, patent class dummies, and total citations received. We find that among patents of the same age, as the number of total citations increases, the expiration rate decreases. This feature confirms the robustness of using dynamic citations as a measure of the economic value of patents. Furthermore, according to the regression estimates, adding an extra citation received to a patent decreases the log of the probability

Table 1: Transfer Rate and Expiration Rate Conditional on Citations Received

A. Expiration Rate as a Function of Total Citations Received

		Total Citations Received				
Age	Unconditional	0	1	10	20	30
5	.1808	.1990	.1920	.1369	.0920	.0607
9	.2839	.3342	.3243	.2427	.1699	.1155
13	.3202	.3981	.3874	.2969	.2124	.1468

B. Transfer Rate as a Function of Total Citations Received

		Total Citations Received				
Age	Unconditional	0	1	10	20	30
1	.0251	.0250	.0254	.0285	.0324	.0369
8	.0182	.0175	.0177	.0199	.0226	.0258
17	.0085	.0073	.0074	.0084	.0095	.0110

of being expired by 0.043 units[9]. The following table presents the predicted expiration rate conditional on the number of total citations received by each renewal date.

Pattern #4: Patents with a higher number of total citations citations received are more likely to be traded. Logit analysis shows that an extra citation received by a patent increases the log of the probability of being traded by 0.011 units. For instance, in Table 1B we show the predicted probability to be traded of active patents conditional on the total citations received. We find that patents with 1 total citations received at age 8 have an estimated probability of being traded of 0.0177 at age 8, this probability jumps to 0.038 if total citations received are 60, and it spikes to 0.063 if total citations received are 100.

Pattern #5: This fact focuses on the effects of the trading decision on the retrading and the future renewal decision of the patent. In Table 2, we show a combination of results that explain how the trading decision and its timing show the difference between previously traded and non-traded patents.

In particular, columns labeled as "Not Previously Traded" and "Previously Traded - Any year" of Table 2 show that among patents of the same age, traded patents are more likely to be retraded and less likely to expire. We see that previously traded patents are twice more likely to be retraded. With respect to the renewal decision, traded patents are about 5 percentage points less likely to expire at each of the renewal dates.

In addition, columns labeled as "1 year" and "4 years" show that these differences are even more striking when we consider the timing at which a previous trade took place. For

[9] Details about the logit analysis can be found in a companion paper: "Measuring the Transfer of Patents," manuscript, at http://www.econ.umn.edu/~carles/research.htm

Table 2: Percentage of Active Patents Traded and Expired

	Age	Not Previously Traded	Previously Traded (Years since last trade)		
			Any Year	1 year	4 years
Expiring Decision	5	18.6	12.6	7.1	14.4
	9	29.2	23.4	11.8	23.5
	13	33.0	28.3	15.6	28.5
Trading Decision	4	2.01	4.47	5.05	-
	8	1.56	3.78	4.85	3.58
	12	1.27	2.62	3.27	2.47

instance, patents traded a year before a renewal date are about half as likely to expire at that renewal date than patents traded four years ago. Finally, patents traded one year ago are twice as likely to be traded than patents traded four years ago.

The next section develops a model that interprets the key patterns.

3 A Model of Patent Trades

There is an extensive work in the theoretical literature on patents. Our earlier work (Serrano, [34]) develops a dynamic model that highlights the consequences of costs of adopting technology in the trading of patents. Here, we allow for the revenue of the patent to evolve stochastically within the firm. Serrano [34] considers deterministic deprecitation.

The starting point for my theory is Schankerman and Pakes [33] and Pakes [29]. They examine the problem of a patent owner deciding in each period whether or not to pay the renewal fee and thereby extend the life of a patent. The contribution of my theory is to introduce into the model, in each each period, an alternative potential owner who may have a greater valuation for the patent than the owner at the beginning of a given period. To transfer a patent to a new owner involves a resource cost, a transaction cost. In summary, whereas Schankerman and Pakes' model has one margin, should the patent owner pay the fee for renewing the patent, my model has a second margin, should the cost of technology transfer be paid to reallocate the patent to an alternative owner.

The economy is populated by a large number of finitely lived patents and a large number of firms. Time is indexed by $a = 1, ..., L$ where $L < \infty$.[10] Initially patents are randomly paired to a fraction of all firms subject to each firm holding at most one patent. The rest

[10] Notice that periods and the age of a patent are interchangeable.

of the firms are potential buyers.

Patents provide a stream of per period revenue to the firms that owns them. [11] Profits are defined as the expected discounted value of such a sequence of per period patent returns x_a minus renewal fees c_a. The role of the renewal fees is to extend patent protection whenever they are paid, otherwise patent returns are zero thereafter. Consequently, we can define the value of a firm as the value of its patent.

The market for patents opens every period. The market allows the ownership of patents to be reallocated across firms. Some firms might be more productive than others in the use of a patent, thus potential buyers can be alternative owners of patents by making succesful acquisition offers. In particular, we assume that there are N offers for every patent at every period. Acquisition offers can be ranked according to the improvement in revenue that they represent with respect to the current owner of a patent. We also assume that the owners of patents consider only the offer with the highest improvement, i.e., g^e. The *extreme* improvement offer represented by $g^e \in [0, B^e]$ is distributed with a cdf F_{g^e}. For simplicity, it is assumed that the owner of a patent makes a take it or leave it offer to the buyer.[12] Thus the revenue of the potential buyer is:

$$y_{a+1} = g_a^e x_{a+1}$$

where F_{g^e} is invariant over a and independent of x.

The diffusion of innovations across the boundaries of a firm is not cost free. An investment must be pursued to make an efficient use of the acquired knowledge. We consider a fixed cost of technology transfer, τ, independent of the age of the patent and the potential gains from trade. The existence of significant costs of technology transfer is a well known fact documented in the literature of intellectual property transfer. Teece [36] is an early reference.

Patent revenues during the patent's life cycle evolve stochastically within a firm. While low intertemporal growth might represent imitation by other firms or the arrival of superior technologies to produce a similar good; high growth might be due to the arrival of new applications that enhance the returns of the innovation, learning about the product market of the innovation, etc. Thus, the revenue in $a+1$ of a patent of age a with current revenue z_a is:

$$x_{a+1} = g_a^i z_a \qquad a \in \{1, ..., L-1\}$$

[11] Licensing of patents is in the background. Licensing affects the per period revenue of a patent but not its ownership.

[12] Alternative bargaining methods do not affect the *qualitative* results of the model.

where $z_a \in \{0, x_a, y_a\}$ and $g_a^i \in [0, B^i]$ is a random variable that represents the growth of internal returns with a cdf:

$$F_{g_a^i}(u^i; a) = \Pr[g_a^i \leq u^i; a]$$

For simplicity, we consider a process where the growth of returns within the firm is independent of their level.[13] Section 4.4 considers the implications of a process of growth of returns that allows for dependence on the level of returns.

The specification of the timing completes the model. At the beginning of every period the owner of every patent considers the best acquisition offer. Firms and potential buyers know the per period return of the patent whether it is allowed to expire, 0; kept, x; or sold,y. Then, firms choose whether to sell, keep or let the patent expire. Once a decision has been made (and the patent reallocated whenever it has been sold), the patent per period revenue $z \in \{0, x, y\}$ is collected. The period ends when the new g^i and g^e that determine next period revenues are known.

Now, we can analize the maximization problem of a firm.

3.1 The Maximization Problem of a Firm

Consider a firm that holds a patent prior to its a^{th} renewal with current revenue x and a potential buyer with revenue y. The value of the patent $\widetilde{V}(a, x, y)$ is the maximum of the value of selling, keeping or let the patent expire.

$$\widetilde{V}(a, x, y) = \max\{\widetilde{V}^S(a, x, y), \widetilde{V}^K(a, x, y), 0\} \qquad a = 1, ..., L$$

where

$$\begin{aligned} \widetilde{V}^S(a, x, y) &= V(a, y) - \tau \\ \widetilde{V}^K(a, x, y) &= V(a, x) \end{aligned}$$

Let $V(a, z)$ be the discounted expected value of a patent with return $z \in \{x, y, 0\}$ at age a when the firm is commited to pay the renewal fee:

$$V(a, z) = z - c_a + \beta E[\widetilde{V}(a + 1, x', y')|a, z]$$

[13]The case for which the growth of returns is independent of their level has a counterpart in empirical industrial organization. Many studies have persistently found evidence for which the growth of firms is independent of their size, known as Gibrat's law. This general case has not been previously considered in the patent literature. However, linear depreciation, which is a particular case of Gibrat's law, has been previously used in the literature on estimating the value of a patent (see Lanjouw, Pakes and Putnam [24] for a survey).

The option value of a patent of age a and current return z is:

$$E[\widetilde{V}(a+1,x',y') \mid a,z] = \int\int \widetilde{V}(a+1,u^i z,u^e u^i z)\mathrm{d}F_{g_a^i}(u^i;a,z)\mathrm{d}F_{g^e}(u^e)$$

where the operator $E[.]$ denotes an expectation conditional on the return of the current owner and the age of the patent, the next period internal returns are $x' = g_a^i z$, and the external returns are $y' = g^e x'$.

For simplicity, it is assumed that the owner of a patent makes a take it or leave it offer to the buyer.[14] Thus, the value of selling $\widetilde{V}^S(a,x,y)$ is also the market price of the patent.

Prior to the description of the equilibrium of the model, several assumptions are needed to characterize basic properties of the value function. Basic continuity conditions for the process of growth of returns within the firm $F_{g_a^i}(u^i;a)$ and the process of gains from trade $F_{g_a^e}(u^e)$ are assumed (i.e., generality conditions to guarantee the continuity and existence of the value function). Moreover, we consider that $F_{g_a^i}(u^i;a)$ is weakly increasing in a, i.e., the expected growth of returns within the firm decreases with the age of the patent.

The following Lemma shows that the value function of a patent is continuous, weakly increasing in the returns of the patent and weakly decreasing in patent age.

Lemma 1 *The value function $\widetilde{V}(a,x,y)$ is continuous and weakly increasing in the current return of the holder of the patent, x, and the return of the potential buyer, y. The option value $E\widetilde{V}(a+1,x',y'|z,a)$ is weakly decreasing in a.*

Proof. See Appendix. ■

4 The Selection and Horizon Effect

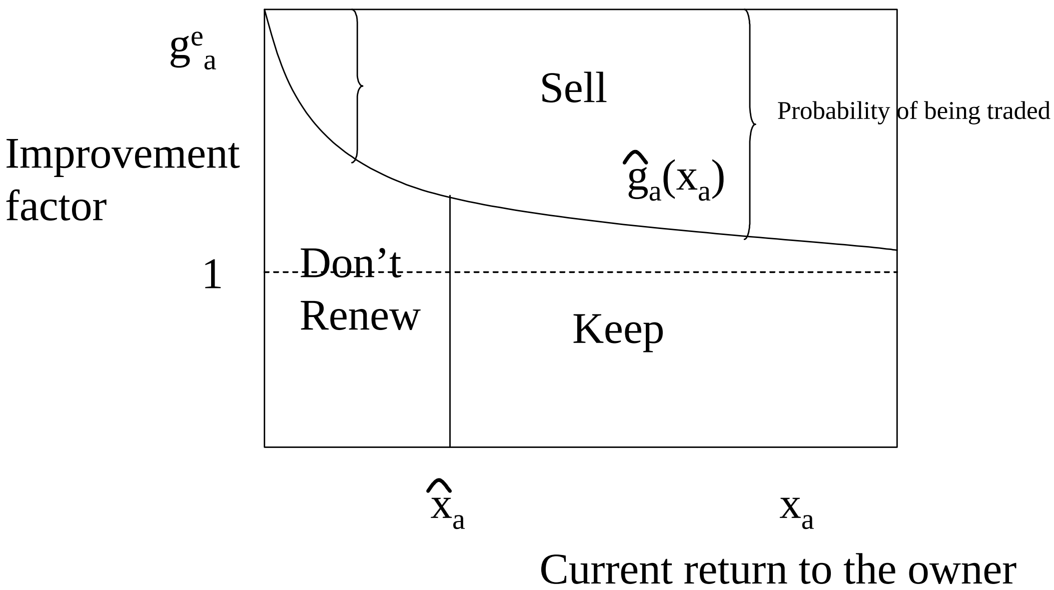
The solution of the problem of the firm can be summarized into two cuttoff rules that describe the policy space: $\{\widehat{x}_a(\tau)\}_{a=1}^L$ and $\{\widehat{g}_a(x,\tau)\}_{a=1}^L$.[15] $\widehat{x}_a(\tau)$ is defined as the patent return that makes the holder of a patent indifferent between keeping or letting a patent of age a expire. $\widehat{g}_a(x,\tau)$, instead, represents the potential improvement in external growth of returns that makes a firm indifferent between selling the patent or not.

The two rules describe three regions in the policy space of patent of age a : E, K, and S. The regions correspond respectively to: let the patent expire, keep it or sell it. Regions E

[14] Alternative bargaining methods do not affect the *qualitative* results of the model.

[15] Both cutoff rules also depend on the states (x,y,a), the renewal fees, c_a, and the cost of technology transfer, τ. However, c_a and y have been omitted.

Figure 2: Policy Space



and K are separated by a straight line $\widehat{x}_a(\tau)$, which is independent of the external growth of returns g^e. For low current patent returns, that is $x < \widehat{x}_a(\tau)$, the firm chooses between not renewing and selling the patent. So, the cutoff $\widehat{g}_a(x, \tau)$ separates the areas E and S. Finally, for sufficiently high returns, that is $x > \widehat{x}_a(\tau)$, letting the patent expire is not an optimal choice, thus $\widehat{g}_a(x, \tau)$ delineates the K and S regions.

The following figure shows the policy space.

The policy space of the model embeds the two mechanisms that characterize the transfer and renewal of patents. First, the cost of technology transfer creates a selection effect so that better patents are more likely to be traded. This selection effect accounts for the discontinuous increase in the transfer rate after renewal, and also for the reason that traded patents are more likely to be retraded and less likely to expire. Second, there is a horizon effect that explains why transfer rates decrease as patents get closer to their expiration date. A shorter horizon implies less time to amortize the cost of technology transfer.

4.1 The selection effect: policy functions for fixed age

This section characterizes the properties of the cutoff rules for a patent of a given age. We prove that the cutoff rule $\widehat{g}^e_a(x, \tau)$ is weakly decreasing in x. The result implies that the probability of an active patent being traded is a weakly increasing function of its current revenue. Moreover, we show that the Pakes and Schankerman patent renewal rule still holds in our extended framework.

The cost of technology transfer creates a selection effect so that patents with higher ex ante per period returns are more likely to be traded. In particular, we show that the level of improvement $\widehat{g}_a^e(x,\tau)$ that makes indifferent a owner of a patent between selling it or not is a weakly decreasing function of the revenue of its patent x. Thus, patents with higher revenue require buyers characterized by lower improvement offers in order to be indifferent between selling the patent or not. Consequentely, the probability that an active patent is traded is weakly increasing in x. The following proposition shows this result.

Proposition 2 *If $\tau > 0$, then (i) the function $\widehat{g}_a(x,\tau)$ is weakly decreasing for all x, (ii) $\widehat{g}_a(x,\tau) > 1$, (iii) the probability of being traded is increasing with x.*

Proof. See Appendix. ■

The selection effect accounts for why patents with higher number of total citations received (i.e., patents with higher value) are more likely to be traded (pattern #4) and less likely to be allowed to expire (pattern #3). Pattern #4 is an implication of Lemma 1 and the monotonicity property proved in Proposition 1. While Proposition 1 show respectively that patents with higher revenue are more likely to be traded to expire, Lemma 1 proves that the value of a patent is increasing in its revenue. Pattern #3, however, holds by the definition of the cutoff rule $\widehat{x}_a$: patents with revenue lower (higher) than $\widehat{x}_a$ are (are not) let to expire. Moreover, the selection effect implies that traded patents are more likely to be traded again and less likely to be allowed to expire (pattern #5). Previously traded patents expire less often and are more likely to be traded because patents they have on average higher revenue than the not previously traded ones.

Another implication of the selection effect is the jumps of the transfer rate immediately after each renewal date. Immediately after each renewal date the average revenue of an active patent increases because patent with the lowest returns were let expired. Thus, the probability of an active patent discontinuously increases after a renewal date because the selection effect implies that patents with higher returns are more likely to be traded on average.

Remarks

This section has shown that patents with higher ex ante returns are more likely to be traded, and its implications are traded patents are more likely to be retraded and less likely to expire. This result had relied on two key assumption of the model: (i) a fixed cost of technology transfer, and (ii) that gains from trade are relative to the current return of the patent.

First, if the cost of technology transfer, instead of a fixed cost, is a proportion of x (i.e., $\tau = \alpha x$, where $\alpha \in (0,1]$), then the result of proposition 1 still holds from any $\beta > 0$. However, if the cost of technology is either zero or proportional to the value of the buyer prior to taxes, $V(a,y)$, then the probability of being traded is independent of the return of the patent (i.e., it is flat with respect to x). So, the implication of these two hypothetical cases would not coincide with the facts observed in the data. Thus, positive costs of technology transfer are necessary to account for the features observed in the data.

Second, if the process of external growth of returns that allows for trades in patents was in levels rather than in growth rates (i.e., the random draw could be a direct return y, rather than a g^e), then we can show that the improvement factor $\widehat{g}_a^e(x,\tau)$ is still weakly decreasing in age but the monotonicity result of the probability of a patent being traded does not necessarily increases with age. In particular, $\widehat{g}_a^e(x,\tau)$ would be U-shaped as a functio of x. We have no empirical evidence dictating that $\widehat{g}_a^e(x,\tau)$ is U-shaped. On the other hand, assuming that the process of external growth of returns implies testable implications that are consistent with the patterns observed in the data.

The next section focuses on the effects of age in the trading and renewal decision of a patent.

4.2 The horizon effect: How do policy functions change with age?

This section studies how the transfer rate of patents varies over their life cycle. In particular, it analyzes the comparatives statistics, with respect to the age of a patent, of the policy functions for a fixed patent return. The interplay between the cutoff $\widehat{g}_a(x,\tau)$ and the age of a patent determines the probability of being traded conditional on survival. This probability might be of interest because it explains the expected efficiency gain from a patent transfer. In addition, the section also looks at how the likelihood to expire changes over the patent's life cycle (i.e., whether the cutoff $\widehat{x}_a(\tau)$ affects the probability of being allow to expire as a function of age).

4.2.1 The transfer rate of a patent during its life cycle

The pattern #1 shows that the transfer rate of patents monotonically decreases as a function of age with the exception of the renewal dates. Immediately after renewal, the transfer rate discontinuously increases.

In the model, there is a horizon effect that explains that transfer rates decrease as the patent gets closer to its expiration date. A shorter horizon implies less time to amortize the fixed cost of technology transfer.

The transfer rate of a patent is the probability of being traded conditional on survival. Patents are traded if efficiency gains from purchasing offers are such that $g^e > \widehat{g}_a(x, \tau)$. So the probability of being traded is just $\Pr[g^e > \widehat{g}_a(x, \tau)]$, which, in particular, depends on a. In order to show that the transfer rate is decreasing with age, it suffices to prove that the cutoff $\widehat{g}_a(x, \tau)$ is increasing in a for a fixed x.

Proposition 3 *If assumption G holds and $\tau > 0$, then for all x and τ, (i) $\widehat{g}_a(x, \tau)$ is increasing in a, and (ii) the probability of being traded conditional on survival is weakly decreasing in age, a.*

Proof. See Appendix. ■

4.2.2 The expiration rate of a patent during its life cycle

This section studies the probability of being expired as a function of the age of the patent. The foundations of this result were shown in Schankerman and Pakes [33] and Pakes [29].

Evidence from U.S. patents presented in pattern #3 shows that the proportion of patents allowed to expire increases with the age of a patent. It is sufficient to show that the cutoff $\widehat{x}_a(\tau)$ is increasing in the age of the patent to account for this feature. The following proposition proves it.

Proposition 4 *If the renewal fees schedule is weakly increasing, then the cutoff $\widehat{x}_a(\tau)$ is weakly increasing in age. Therefore, the probability of being allowed to expire weakly increases as a function of patent age.*

Proof. See Appendix. ■

More interesting, however, are the effects of the renewal decision on the distribution of patent returns as a function of patent age. This is particularly relevant when the number of renewal dates is low, as it is in the U.S. patent system. In this case, the number of patents that might expire in each of the renewal dates can be substantially large. Thus, immediately after a large fraction of patents expires, the average patent return increases. The next section develops and explores this result in-depth.

4.3 A horse race: The selection compared to the horizon effect

Renewal dates and their implications into the trading decision are interesting events on which to focus. These events link the results of the model with the data.

The model predicts that immediately after renewal, the selection and horizon effects display opposite trends towards the probability for a patent of being traded. On the one hand, right after the renewal date, the average patent return increases, so patents are more likely to be traded according to the selection effect. On the other hand, the horizon effect implies that as the age of a patent increases, patents are less likely to be traded . Therefore, there exists a horse race between the two effects. The race determines the observed proportion of patents that are being traded. Therefore, the model helps to disentangle the forces under the selection and horizon effect.

The selection effect is sufficiently stronger than the horizon effect as presented in pattern #1. This pattern shows that the probability of an active patent being traded discontinously increases in all renewal dates and then monotonically decreases until the next renewal date. Thus shortly after a renewal date the selection effect is stronger than the horizon effect, but a patent ages the horizon effect plays a leading role in characterizing the trading of patents.

5 Estimation

In this section, we discuss the estimation and identification of the parameters of the stochastic specification of the model. The fundamental basis of the identification strategy relies on the interplay of the cost of technology transfer with the age and revenue of a patent.

5.1 Stochastic specification of the model

The stochastic specification that is estimated contains seven parameters. We set the discount factor $\beta = 0.9$ as in Pakes [29]. The rest of the parameters are jointly estimated.

Patents are initially granted to a fraction of firms. We focus on the market for patents and for the sake of tractability consider the invention decision exogenous. Thus, the initial revenue of an innovation is distributed lognormally:

$$\log(x_1) \sim F_{IR}(\mu, \sigma_R)$$

where $F_{IR}(.)$ is a normal distribution.

The revenue of the best potential buyer for a patent is determined by the process of gains from trade. We consider that the process g^e is independent of the revenue of a patent

x, the age of the patent and the process g_a^i.[16] Thus,

$$y_a = g^e x_a$$

where g^e follows an exponential distribution with parameter σ^e

$$F_{g^e}(u^e) = 1 - \exp(-\frac{u^e}{\sigma^e})$$

In addition, for simplicity we consider that the patent revenue from period to period either depreciate at a positive fixed factor rate δ with probability θ or drops to zero with probability $1-\theta$. The process illustrates both the possibility that patent per period returns decreases continuously during the life of patent or drastically drops to zero. It aims to capture the effects of imitation or the the arrival of superior competing technologies. Thus next period returns of the owner of a patent are

$$x_{a+1} = g_a^i * z_a$$

where $g_a^i \in \{\delta, 0\}$ and $z_a \in \{x_a, y_a, 0\}$ is the per period return if the patent is respectively, kept, sold and allowed to expire.

Finally, the estimation specification allows for transactions of patents that are random events. Random events understood as transactions that are recored as such but do not involve neither a cost of technology transfer not a gain from trade. Such transfers might occur in every period with probability ε. We permit random trades in the estimation to account for potential sources of 'noise' in the data. We understand the conservative nature of the assumption in that the estimates will imply a lower bound on the gains from trade in the market for patents.

To sum up, the complete stochastic specification of the model contains seven parameters, $w = (\mu, \sigma_R, \sigma^e, \tau, \varepsilon, \theta, \delta)$.

5.2 Estimation strategy and estimates

The parameters of the model are structurally estimated using the simulated generalized method of moments. This method involves finding parameters w so that they minimize the distance between the empirical moments, defined as those from the data, and the simulated moments generated by the model.

[16] In practice, we define the age of a patent a as the number of years from its grant date because patent renewal fees are due by the end of the 4^{th}, 8^{th} and 12^{th} year from their grant date.

The moments generated by the model are simulated because they cannot be solved analytically due to the structure of the model. In particular, our estimation strategy consists in fitting the proportion of active patents that are traded conditional on being previously traded and not previously traded (32 moments), the proportion of active patents allowed to expire conditional on having been previously traded and not previously traded (6 moments). In total there are 38 moments. The following algorithm explains the details of the procedure.

5.2.1 Estimation algorithm

A simulating procedure was first applied in a patent renewal model by Pakes [29]. Pakes used a maximum likelihood estimator. Lanjouw [23] , instead, applied the simulated generalized method of moments as we use here.

To estimate the parameters of the model we find the simulated minimum distance estimator[17], $\widehat{w}_N$, of the true k parameter vector, w_0[18], and N is the sample size.

$$\widehat{w}_N = \arg\min_{w} v_j ||h_N - \eta_N(w)||$$

The vector h_N is defined as the sample hazard proportions or empirical moments, and the vector $\eta_N(w)$ as the ones that are simulated. In particular, the vector of empirical moments contains the following. First, the probability to expire of active patents conditional on previously traded, the probability to expire of active patents conditional on not previously traded. A vector η_N^{ε} of 6 moments. Second, the probability to be traded of active patents conditional on previously traded and the probability to be traded of active patents conditional on not previously traded. A vector η_N^{φ} of 32 moments. In summary, the vector $\eta_N(w) = [\eta_N^{\varphi}, \eta_N^{\varepsilon}]$ contains 38 moments.

The vector w_0 is defined as the unique solution to the equation

$$G(w) = \sum_{j=1}^{38} v_j \left(\frac{\eta_j(w) - \eta_j}{\eta_j} \right)^2 = 0$$

where η is the vector of the true hazard probabilities, $\eta(w)$ are the hazards predicted by the model with paramete vector w, and the v_j are the elements of the diagonal of the weighting

[17]The capital letter N denotes the sample size.

[18]Pakes and Pollard (1989) have showed conditions under which $\widehat{w}_N$ converges to w_0, and $\sqrt{n}(\widehat{w}_N - w_0)$ satisfies a central limit theorem.

matrix. The weighing matrix is defined as

$$v(w) = \text{diag}[\sqrt{n/N}]$$

where $n = [n_1, ..., n_j, ..., n_{32}]$, and n_j is the number of patents in which moment j is conditioning on. Note that the metric of the distance we choose is the proportional distance between the empirical and simulated moment.

Using the simulated minimum distance estimator requires to simulate the moments $\eta_N(w)$ and to minimize the distance between these moments and their empirical counterparts.

Given a vector of parameters w, the simulated moments $\eta_N(w)$ are generated in the following way. First, we solve the model recursively. This step involves calculating the value function of a patent at age a at a number of selected grid points. Next we find the cutoff rules $\widehat{x}_a$ and $\widehat{g}_a(x)$ as function of w and c_a. In order to calculate the value function at age $(a-1)$, we approximate the integral that defines the option value of a patent with quadrature approximation methods. Note that some of the limits of the integral will be defined as a function of the cutoff rules $\widehat{x}_a$ and $\widehat{g}_a(x)$.

Second, we calculate the simulated moments as the average obtained from S simulated populations of N patents. Each simulated population of patent consists of taking pseudo random draws from the distribution of initial returns F_{IR}, and then we pass each initial patent return through the stochastic process of returns of the model implied by distribution of internal growth of returns $F_{g_d^i}$ and the distribution of external returns F_{g^e}, etc.

Finally, we average all simulations and calculate the simulated moments $\eta_N(w)$[19]. To minimize the distance between the empirical and the simulated moments we use a minimization algorithm based on simulated annealing methods.[20]

5.2.2 Estimates

The estimates are presented in Table 3[21].

[19]We run 15 simulations, and in each simulation we have 453,683 patents with their respective draws of initial returns, internal and external growth of returns.

[20]In particular, we use simulating annealing methods. In particular, we use the simulated annealing algorithm developed and tested in Goffe, Ferrier and Rogers [15].

[21]We use a bootstrap method to obtain the standard errors of the parameters estimates of the model. In particular, we do the following. We take the parameter estimates and calculate the simulated moments generated by the model. Then, we draw a S simulations of N patents, and apply the estimating procedure as if the newly simulated moments where the empirical moments. We repeat this procedure 10 times in order to simulate the distribution of parameter estimates.

Table 3: Estimates

Description (Parameter)	Estimate
Depreciation factor (δ)	0.9316
Full Depreciation (θ)	0.9556
Mean parameter of the Lognormal Initial Distribution (μ)	8.2421
Std. Deviation parameter of the Lognormal Initial Distribution (σ_R)	1.1587
Cost of Technology Transfer (τ)	12,893
Mean External Growth of Returns (σ^e)	0.3839
Random Trades (ε)	0.00618

5.2.3 Identification

The cost of technology transfer is jointly identified by the difference in the probability of a patent being traded conditional on having been previously and not previously traded, the probability of a patent being allowed to expire conditional on having been previously and not previously traded. Moreover, the curvature of the probability of an active patent being traded conditional on having been previously traded or not as well as the size of the jumps of the the transfer rate also play an important role in the identification of the costs of technology transfer. In summary, the higher the costs are the larger are the differences between previously traded and not previously traded in the probability of an active patent being traded (or being allowed to expired).

For the sake of explanation, let us start considering the case in which the transaction cost is zero. Zero costs of technology transfer imply no selection and no horizon effect. Thus, there are no significant differences between traded and no traded patents in the simulated probability of a an active patent being traded and being expired. In addition, the probability of an active patent being traded is flat over the life cycle of a patent except at the renewal rates. On the other hand, if we consider increasing the costs of technology transfer, then the difference between probability of an active patent being traded conditional having been previously traded or not also increases; as weel as the difference between the probability of an active patent being allowed to expire conditional on having been previously traded or not.

Similarly to the previous literature estimating the value of patents, the main source of identification of their Dollar value is the schedule of renewal fees. Moreover, there is no information in the data we currently use in the estimation that identifies an upper bound of the value of patents that are renewed at all renewal dates. Thus, the fact that owners allow their patents to expire indicate that the present value of their future expected patent returns is below the one of the renewal fee. Obviously, it is being assumed the fact that owners of patents are willing to pay renewal fees expecting that their future returns will be high enough to compensate these costs.

The parameters of the process of internal growth of returns, (δ, θ), and the ones from the initial returns, (μ, σ_R), are jointly indentified from the proportion of active patents that expire conditional on having been previously traded or not, and the proportion of active patents that are traded conditional having been previously traded or not. The parameter δ is identified in part from the expiration rate and the curvature of the transfer rate as patents get closer to their expiration date. Moreover, the parameters μ, σ_R are identified jointly from the moments that relate to the expiration decision and also from the level of the transfer rate conditional on previously traded or not, especially early in the life of a patent

The parameter ε is jointly identified from both the level of the proportion of active patents that expire conditional on having been traded, and the proportion of active patents that are traded conditional on having been previously traded. A higher ε implies that a higher proportion of active and previously traded patents will expire, and a lower proportion of active and previously traded patents will be retraded.

Finally, the parameter σ^e is identified from the level of the probability of an active patent being traded. The higher the probability is, the higher σ^e is too. For the sake of tractability, we assume that the cdf F_{g^e} was independent of age. As a matter of fact, we could also allow for age depence. Notice that the costs of technology transfer are mainly identified from the differences of previously and not previously traded patents in the probability of an active patent being allowed to expire and the probability of an active patent being traded. We argue that age in the dependence in the process of external gains from trade can be indentified through the curvature (as function of age) of the the probability an active patent being traded conditional on having being previously traded ant not previously traded.[22]

6 Estimation Results

This section discusses the fit of the model and quantifies what are the gains from trade in the market for patents.

6.1 Fit of the model

An indicator of how the estimated model fits the data is to compare the empirical moments and the simulated moments from the model. We show that the model fits reasonably well

[22]For instance, we could consider a function form such as: $F_{g^e}(x) = \sigma^e \phi^{a-1}$ where $\phi > 0$.

Table 4: Fit of the Expiration Rate Conditional on Previously or Not Previously Traded

A. Number of Patents Allowed to be expired as a Percentage of Previously Traded Active Patents

Age of the Patent (Years)	Model	Data
5	13.46	12.57
9	19.20	23.39
13	21.16	28.35

B. Number of Patents Allowed to be expired as a Percentage of Not Previously Traded Active Patents

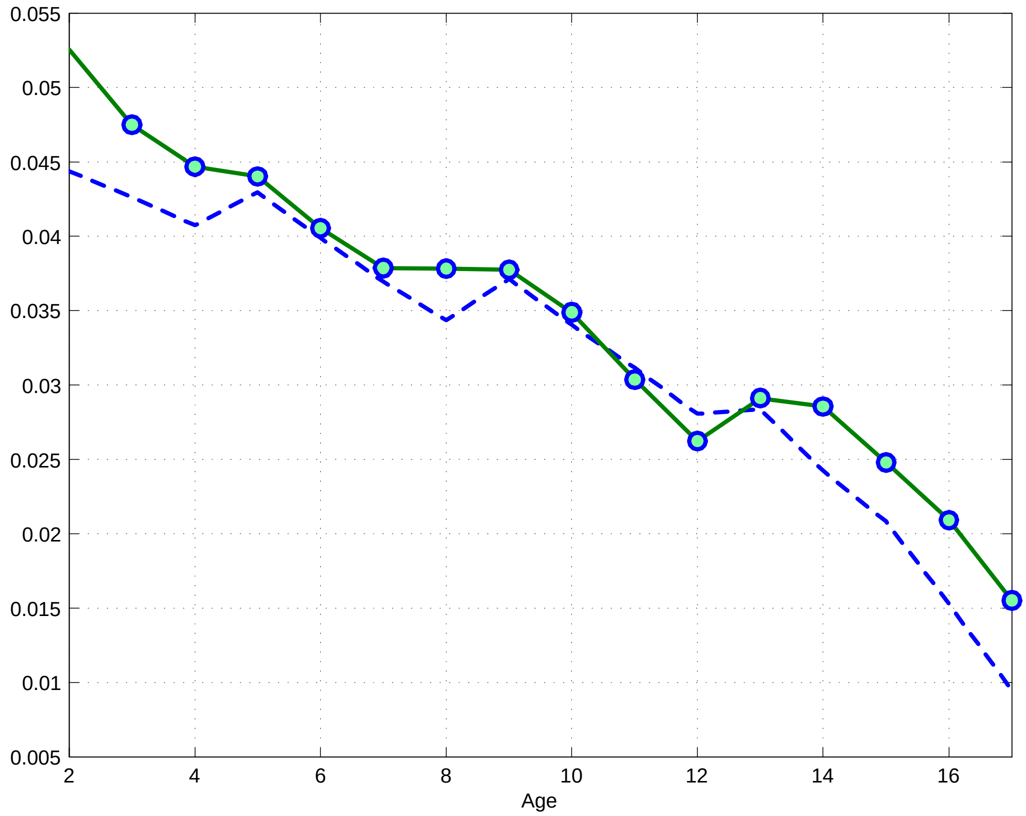
Age of the Patent (Years)	Model	Data
5	19.22	18.62
9	29.83	29.25
13	34.52	33.01

the data moments we aim to match. In Table 4, we present the number of patents allowed to be expired as a percentage of traded and not traded active patents. We see that both the empirical and the simulated expiration rate are increasing as a function of the age of the patent. The model, however, tends to predict a lower steepness in the moment that relates to the probability of an active patent being expired conditional on having been previously traded. Intead, the moment of the probability of an active patent being expired conditional on not having been previously traded is remarkably well fitted.

Moreover, Figure 3 and Figure 4 present respectively, the number of patents that are traded as a percentage of active previously traded and not previously traded patents. Figure 3 shows that the model is able to capture the decreasing shape of moments to be matched, including the discountinuous jumps of the transfer rate immediately after each renewal date and the sharp decrease when patents get closer to their expiration date. Nevertheless, the simulation generated by the model tend to overpredict the jumps of the transfer rate. We conjecture that this feature might be as a result of the timing in which the trading decision occurs in the model as the time in which occurs in practice. In the model, choices are made at the beginning of the period, while in practice these decisions take place continuously within a year. In addition, Figure 4 also shows that the model fits well the decreasing pattern of the data and the jumps of this statistic observe immediately after a renewal date.

An additional measure of how the estimated model fits the data is assessing the predictions of the model in other moments that the ones used in the estimation strategy. For instance, we can us the transfer rate, that is the the number of traded patents as a percentage of all active patents. For instance, Figure 5 show the fit of the model generating this moment and the one from the data. We see that the fit is good.

Figure 3: The Probability of an Active Patent Being Traded Conditional on Previously Traded



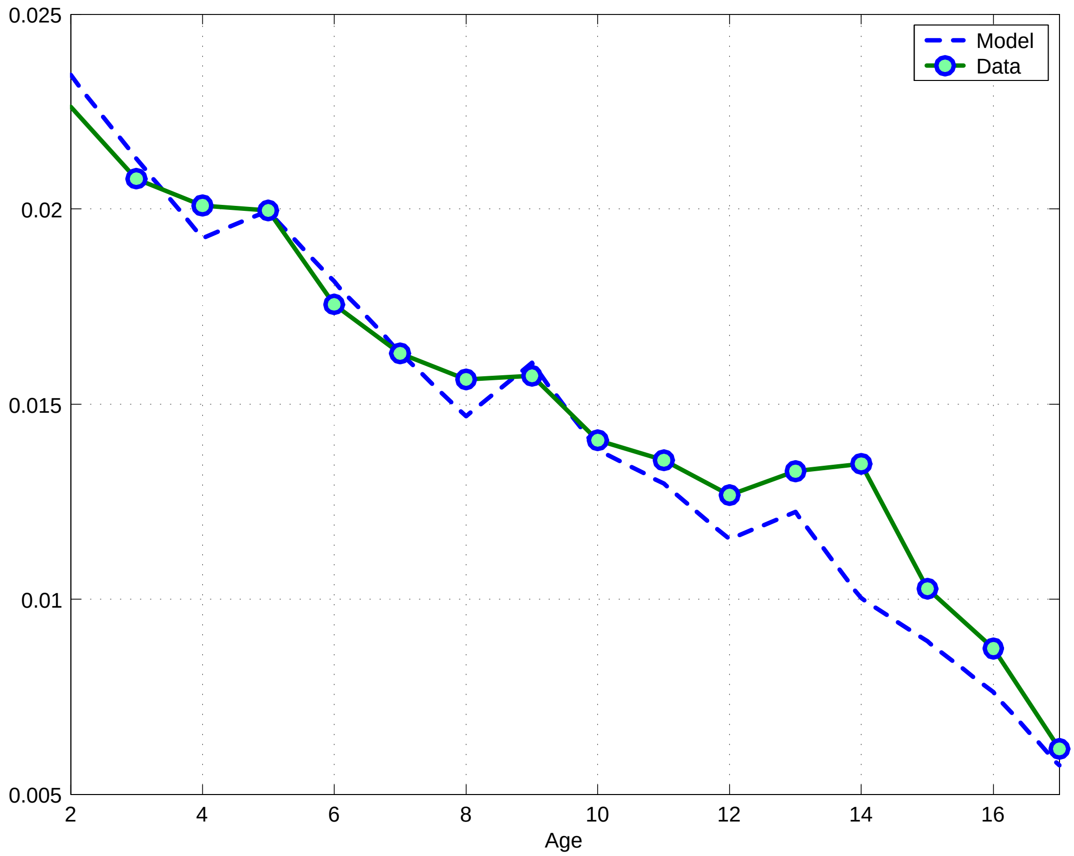
We also present the fit of data moments that are not included in the estimation strategy. In particular, we look at the number of patents that are allowed to expire at a renewal date as a percentage of all active patents. In Table 5 we present the expiration rate in the data and the one generated by the model. The model does a remarkable job in fitting these moments.

6.2 Discussion of estimated parameters

The parameter estimates are presented in Table 3.

Initial returns The parameters μ and σ_R determine the initial distribution of the revenue of patents. A high σ_R implies a large heterogeneity in revenue among patents. A low μ implies that initially the returns of patents are low. For instance, according to the estimated parameters, 4 percent of all patents have per period returns at age 1 below US$ (2003) 500, and the median patent has per period returns US$ (2003) 3815 at age 1.

Figure 4: The Probability of an Active Patent Being Traded Conditional Not Previously Traded



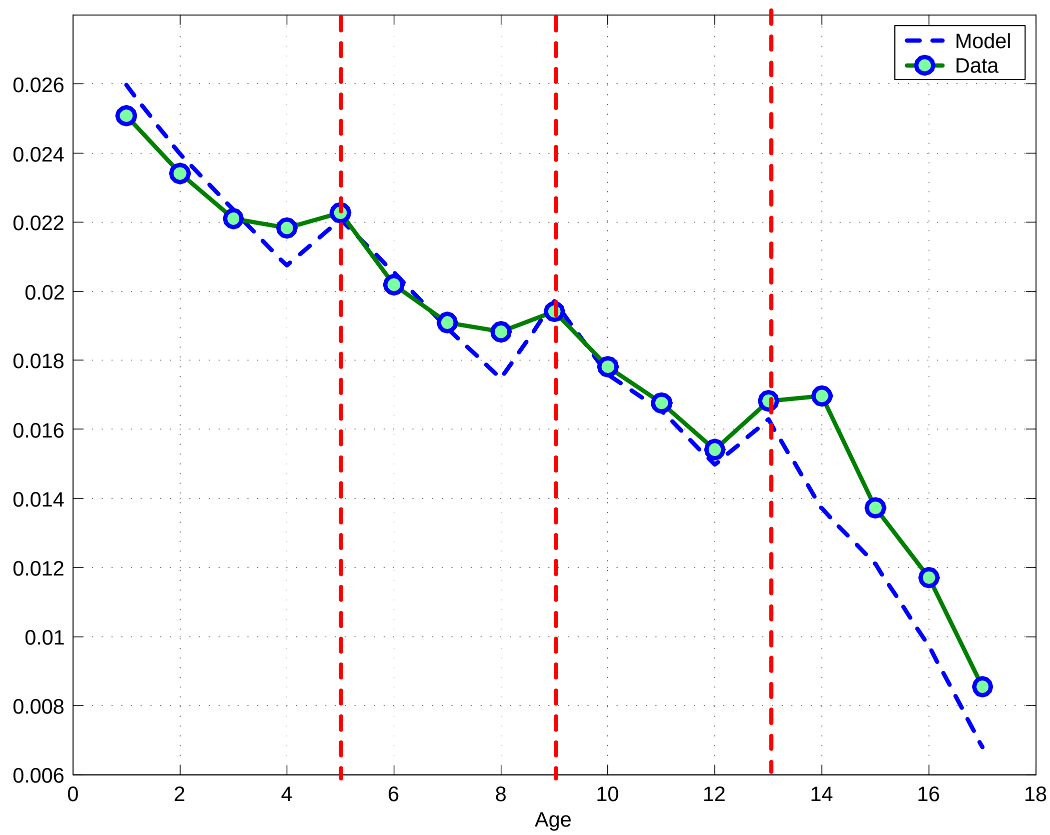
Internal growth of returns The parameters δ and θ determine the process of internal growth of returns. [23]

They determine the full rate of depreciation. For instance, an estimate of $\delta = 0.93$ and $\theta = 0.95$ imply that the per period return of a patent depreciate on average at the rate of 11.65 % a year. This suggests that either competing technologies or imitation erode fast the profits from the protection of intellectual property.

External growth of returns The parameter σ^e of an exponential distribution describes the process of potential gross growth of returns. A high σ^e implies both that is more likely that potential buyers are might be more efficient and that the heterogeneity among efficiency gains is also greater. The parameter is estimated to be $\sigma^e = 0.3839$. This suggests that the probability that in a given period a potential buyer has larger per period returns is

[23] Pakes (1986) and Lanjouw (1998) report similar results for the case of German patents.

Figure 5: The Probability of an Active Patent Being Traded



0.073. However, since the cost of technology transfer is positive, then the decision whether a patent is transferred ultimately depends on both the current per period patent return of its owner and the cost of technology transfer.

Costs of technology transfer The estimate of the cost of technology transfer is US$ (2003) 12,893. Thus, costs of adopting technologies developed by other firms require significant expenditures representing about 12.8 percent of the average value of a traded patent. These expenditures account for the restructuring firm's organization to efficiently use the acquired technology, personel, R&D, etc.

Random trades The parameter ε accounts for patents recordations that are not as a result of the process of gains from trade and cost of technology transfer. The estimate of this parameter is 0.006, i.e., 0.6 percentage points of the proportion of active patents that are traded in every period cannot be explained with the specification of the transfer of technology that we have used. This accounts approximately for 24% of the trades of the

Table 5: Expiration Rate

Age	Data	Model
5	18.99	19.06
9	28.30	28.20
13	30.81	30.74

Table 6: Distribution of the Value of Patents at Age 1

Percentile	Value (US$ 2003)	Cum % of Total
50	17,395	10.2
75	41,291	28.2
80	50,841	34.3
90	87,597	51.8
98.5	257,321	82.9
99.8	599,076	95.2
Mean value	37,767	

patents of age 1, 25% at age 2, 29% at age 5, 35% at age 10, 37% at age 13, 50% at age 15. and 90% at age 17. Thus, since most of the patents are traded early in their life, the model counts as real trades more than 30% of all patents recorded.

Distribution of the value of patents In this section the parameter estimates are used to simulate the distributions of the value of patents and show how this distribution evolves as patents become older. These distributions were calculated by generating a five simulations of 453,683 patents with their respective initial returns, and consequentely following their revenue using the estimated process of internal and external growth of returns while taking into account whether patent were renewed or traded. The value of a patent is obtained as the discounted present value of its stream of patent returns at a given age.

The simulations show that the distribution of patent returns is quite skewed. Table 6 presents some percentiles of such distribution with the estimated value of patents at their grant date. In particular, the value of the median patent is estimated to be US$ (2003) 17,395, but the bottom 50 percent of all patents only account for the 10.2 percent of the total value of all patents in the cohort.

In addition, the distribution of patent returns becomes more skewed as function of the age of the patents. The model predicts that low return patents are less likely to be traded

Table 7: Distribution of the Value of Patents at Age 4

Percentile	Value (US$ 2003)	Cum % of Total
50	9,542	5.6
75	28,116	21.6
80	35,841	27.6
90	65,785	45.7
98.5	207,285	80.0
99.8	449,445	94.2
Mean value	26,667	

and they mainly depreciate either at a rate δ or their returns become zero with probability $1-\theta$. We find that patent with higher per period returns are more likely to be traded, so their per period returns on average depreciate less fast than the ones that are not traded. The trading effect implies, in particular, that the distribution of patent value becomes skewer as a function of their age. For instance, in Table 6 we show that the median patent at age 1 has a value of \$17,385 and the 99.8 percentile is \$599,076. However, in Table 7, which looks at age 4, the median patent is \$9,542, while the 99.8 percentile is 499,445. Thus, the distribution of patent value at age 4 is more skewed than at age 1.

Moreover, we find that the average present value at age 1 of a patent that is eventually traded is \$100,041, and the average value of one that is not traded ever is \$29,660. Thus, traded patents on average are about three times more valuable than their non-traded counterparts.

6.3 Gains from trade

Patents are traded because some firms are more productive than others in the use of a given patent. Consequently, upon the transfer of a patent, gains from trade are realized. In the model, we define the gains from trade as the value of patent rights generated by the ability to transfer patents between firms.

We calculate the value of the gains from trade by comparing the value of patents with and without an active market. In practice shutdown the market involves setting the costs of adopting technology high enough so that no patent is actually traded. We obtain that the market for patents represents 14.2 percent of the value of a traded patent.[24]

In addition, we also show the gains from trade of an average patent. That is a 7.37

[24]The gain from trade is calculated as $\frac{100{,}041-87{,}595}{100{,}041}$.

percent. These gains are obviously smaller than an average traded patent because a large number of patents have low value and patents with low value are less likely to be traded.[25]

This estimate is a lower bound of the gains from trade in the market for intellectual property. We have assumed complete information and an efficient allocation of property rights. Thus, we are underestimating the actual gains trade in this market. In addition, patents might also be licensed, which could potentially imply even more gains from trade. Unfortunately, we cannot account for gains from trade due to licensing because there is not such systematic data available: Licensing transactions tend to be private agreements between firms.[26].

7 Conclusion

This paper is the first work that estimates the gains from trade and the costs of adopting a technology in the market for patents. We find that the market for patents accounts for 14.2 percent of the value of a traded patent. This number is a lower bound of the gains from trade in the market for intellectual property because, in particular, licensing opportunities have not been considered in this accounting. Moreover, the costs of adopting a technology are significant. They represent about 12 percent of the average value of a traded patent. Thus the paper shows how much the market for trading patents increases efficiency in the use of patents as they are reallocated to the most productive firms.

The identification strategy of the cost of technology transfer and the gains from trade rely on the interplay between the cost of technology transfer, the revenue and age of a patent. First, the cost of technology transfer creates a selection effect so that better patents are more likely to be traded. This explains the discontinuous increase of the transfer rate after the renewal decision, and the evidence that traded patents are more likely to be traded and less likely to expire. Second, there exists a horizon effect that explains that the transfer rate decreases as the patent gets closer to its expiration date. This is because the shorter horizon implies less time to amortize the cost of technology transfer. This accounts for the observed decreasing transfer rate over the life cycle of patents. Thus, while the cost of technology transfer is identified by the difference between the probability of an active patent being traded (expired) conditional on having being previously traded or not and the

[25] The gain from trade is calculated as $\frac{37{,}767-34{,}982}{37{,}767}$. Note that the gains from trade are net of taxes and the cost of technology transfer.

[26] Note that if the cost of technology transfer was equal to infinity, no licensing would occur either. Licensing a technology also involves a cost of adoption. What might be ocurring in the market is that the size of the cost of adopting a technology might determine whether technologies are either licensed or sold (assigned).

curvature the transfer rate; The parameter of the process of external growth of returns is identified by the level of the transfer rate.

The parameters of the model are estimated using the simulated general method of moments. In particular, we aim to fit the probability of a patent being traded conditional on having being previously traded, the probability of a patent being traded conditional on having being not previously traded, the probability of a patent being let to expire conditional on having being previously traded, and the probability of a patent being let to expire conditional on having not being previously traded. The model fit of the empirical moments reasonably well.

This work opens new avenues of research. Perhaps most interesting would be to study the sources of innovation and to characterize who are the buyers and sellers of technology. In particular, to trace the flow of technology transfer, and to analyze whether small firms specialize in innovating and then selling their inventions to larger firms, which might have a comparative advantage in their management. Second, to evaluate to what extent the move toward higher protection of patent rights that occurred in the 1980's has facilitated specialization, and consequently trade in patents. Third, to study the transfer of technology over the business cycle. Lastly, this work can also be extended to examine alternatives to promote innovation such as lower taxation on intellectual property transfer.

These questions have not been previously addressed empirically due to a lack of data on how intellectual property assets are traded.

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Appendix: Level dependence in the growth of returns within the firm

Previous sections of the paper have focused on the case for which the growth of patent returns within the firm was independent of the level.

However, the existing patent literature that has estimated the value of patents, such as Lanjouw and Pakes, have considered variations of a process of growth of internal returns that also depends on their level. The following assumption defines an explicit stochastic process of growth of returns within the firm with level dependence.

Assumption L: The random variable g_a^i is distributed with a function that depends on the return z_a and patent age a.

$$F_{g_a^i}(u^i; z_a, a) = \left\{ \begin{array}{lll} 0 & \text{if} & u^i < \delta \\ \Pr[g_a^i \leq u^i; z_a, a] & \text{if} & u^i \geq \delta \end{array} \right\}$$

such that the function $F_{g_a^i}(u^i; z_a, a)$ is increasing in z.

If $F_{g_a^i}(u^i; z_a, a)$ does not depend on age, then the process is defined as constant learning (LC). If $F_{g_a^i}(u^i; z_a, a)$ is increasing in a, then the process is defined as diminishing learning (LD). An extreme case of LD, let is call it LDE, is one in which there exist an $\overline{a} < L$ such that $\Pr[g_a^i = \delta; z_a, \overline{a}] = 1$.

This assumption considers a process such that in every period returns either depreciate at a rate $\delta \in (0, 1)$ or grow at a rate $g^i > \delta$.

The term learning has been previously used in the literature on estimating the value of a patent. Learning allows for the possibility of new opportunities or applications that enhance the returns of a patent to be discovered (i.e., so that their growth rate can be higher than depreciation).

The following proposition proves that, if the internal growth of returns also depends on the level as in the learning process specified in assumption L, then $\widehat{g}_a(x, \tau)$ is not necessarily increasing for all a. In particular, it states sufficient conditions so that $\widehat{g}_a(x, \tau)$ is weakly decreasing for sufficiently young patents. The main implication is that the probability of being traded is hump-shaped as a function of the age of the patent.

Lemma 5 *If the support of the random variables g_a^i and g^e is bounded above, then* $\left|\frac{\Delta V(a+1, x', y'|z)}{\Delta a}\right| \leq Q$.

Proof. See Appendix. ■

Proposition 6 *Let $\overline{a}$ be the patent age at which learning completely vanishes. If assumption LDE, the support of the random variables g_a^i and g^e is bounded above, and the*

$\Pr[g_a^i \leq u^i; z_a, a]$ *is sufficiently concave with* a*, then there exists an age* $a^* < \overline{a}$ *such that for* $a < a^*$ *the function* $\widehat{g}_a(x,\tau)$ *is weakly decreasing in* a*. In addition, if* $a \geq a^*$*, then* $\widehat{g}_a(x,\tau)$ *is weakly increasing in* a*. Therefore, the probability of being traded conditional on survival is hump-shaped as a function of the age of a patent.*

Proof. See Appendix. ■

The rational of this proposition is that if the probability of learning decreases sufficiently fast with age and is also decreasing in the return of a patent, then the option value of keeping a patent with low return experiences larger proportional losses than patents with high returns. If patent returns are low, then learning possibilities initially explain a large part of the option value of a patent because learning is more likely for patents with low current returns. Instead, if current returns are high (for instance, upon a potential trade), then learning, although still important, is less weighted in the option value of a patent because newer profitable applications are less likely. Consequently, if learning fades sufficiently fast as age increases, then the option value of a patent with lower current returns experiences larger proportional losses on average than a patent with higher current returns. Thus, given a fixed efficiency gain, patents that were not traded when very young might be traded when they are slightly older. So, $\widehat{g}_a(x,\tau)$ is weakly decreasing in a. However, as time passes the learning effect vanishes and then the process of returns converges to a scenario in which returns depreciate at a fixed factor. Thus, the effect of trading soon surely dominates again. Then, the function $\widehat{g}_a(x,\tau)$ is weakly increasing in a, and the probability of being traded is weakly decreasing in a. Therefore, $\widehat{g}_a(x,\tau)$ is U-shape, which implies that the probability to be traded conditional on survival is hump-shaped with age.

Nevertheless, a learning process by itself does not necessarily imply that $\widehat{g}_a(x,\tau)$ is U-shaped in a. The previous proposition illustrates the necessity of strong assumptions to show that $\widehat{g}_a(x,\tau)$ is not always weakly decreasing in a. Diminishing learning guarantees that the slope of the option value of a patent for the seller is steeper, however it might be the case that it has less steepness than the one of the buyer. In fact, if learning does not diminish fast enough, then the horizon effect dominates and consequently $\widehat{g}_a(x,\tau)$ is increasing in a.

As a matter of fact, we can show that if the process of learning is independent of the age of the patent, then $\widehat{g}_a(x,\tau)$ is weakly increasing in a. This means that the probability to be traded conditional on survival is weakly decreasing. The following proposition shows this result.

Proposition 7 *If* $F_{g_a^i}(u^i; z, a)$ *is independent of* a*, then (i) the function* $\widehat{g}_a(x,\tau)$ *is weakly increasing in* a*. And (ii) the probability of being traded is weakly decreasing in the age of* *the patent.*

Proof. See Appendix. ■

Appendix: Proofs and Tables (Incomplete)

Proof of Lemma 1:

The proof is an extension based on Pakes (1986) results.

Proof of Proposition 2:

For convenience of notation, let us rewrite the value function $V(a,x,y)$ as $V_a(x,y)$. I want to show that $\widehat{g}_a(x,\tau)$ is weakly decreasing with respect x. I have divided the proof into two parts. The first one studies the case where $\widehat{g}_a(x,\tau)$ is defined as the external growth of returns that makes a firm indifferent between selling and allow the patent to expire. This result is straightforward. Formally the function $\widehat{g}_a(x,\tau)$ is defined as

$$V(a,\widehat{g}_a(x,\tau)x) = 0$$

From Lemma 1 we know that the value function is weakly increasing in per period returns of a patent. Let us suppose that x increases, it must be the case that $\widehat{g}_a(x,\tau)$ is decreasing with x to keep the equality holding.

The second part of the proof analyzes the subtle case in which $\widehat{g}_a(x,\tau)$ is defined as the external growth of returns that makes a firm indifferent between selling and keep the patent. Consider assumption G, that states the internal growth of returns is independent of the level. Patent returns evolve over time according to

$$\begin{aligned} g_a^i &= \frac{x'}{z} \\ g^e &= \frac{y'}{x'} = \frac{y'}{g^i z} \end{aligned}$$

where in the case of assumption G, the joint density function of the random variables g_a^i and g^e, defined as $f_a(g^x, g^y)$ depends upon a but not z.

The argument of the proof is by induction on the age of the patent. Let us start by considering the last period, $a = L$.

$$\begin{aligned} \left[\hat{g}_L(x,\tau) - 1\right] x &= \tau \\ \hat{g}_L(x,\tau) &= \frac{\tau}{x} + 1 \end{aligned}$$

which is decreasing in x.

Now, assume true for $a' > a$, I will show is also true for a.

The decision of whether to sell or keep relies on the following expression.

$$\begin{aligned}
& \widetilde{V}_a^S(x,y) - \widetilde{V}_a^K(x,y) \\
= \; & V_a(y) - \tau - V_a(x) \\
= \; & V_a(g_a^y x) - \tau - V_a(x) \\
= \; & (g_a^y - 1)\, x - \tau \\
& + \beta \left[E\left[\widetilde{V}_{a+1}(x', y') | a, g_a^y x \right] - E\left[\widetilde{V}_{a+1}(x', y') | a, x \right] \right]
\end{aligned}$$

It is sufficient to show that the above is weakly increasing in x. The first term is increasing in x. Now look at the second

$$\begin{aligned}
& E\left[\widetilde{V}_{a+1}(x', y') | a, g_a^e x \right] - E\left[\widetilde{V}_{a+1}(x', y') | a, x \right] \\
= \; & \int_{g_{a+1}^x} \int_{g_{a+1}^y} \left[\begin{array}{c} \widetilde{V}_{a+1}(g_{a+1}^i g_a^e x, g_{a+1}^e g_{a+1}^i g_a^e x) - \\ \widetilde{V}_{a+1}(g_{a+1}^i x, g_{a+1}^e g_{a+1}^i x) \end{array} \right] f_a(g^i, g^e) d g_{a+1}^i d g_{a+1}^e
\end{aligned}$$

For general a, recall that as an induction hypothesis we assumed

$$\widetilde{V}_{a+1}^S(x,y) - \widetilde{V}_{a+1}^K(x,y)$$

was weakly increasing in x. It suffices (given conditions on the joint density funtion of the growth of returns) to prove that the interior of the double integral is increasing in x.

$$\widetilde{V}_{a+1}(g_{a+1}^i g_a^e x, g_{a+1}^e g_{a+1}^i g_a^e x) - \widetilde{V}_{a+1}(g_{a+1}^i x, g_{a+1}^e g_{a+1}^i x)$$

There are four cases to study.

1. in K region with $(g_{a+1}^x x, g_{a+1}^e g_{a+1}^i x)$, in K region with $(g_{a+1}^i g_a^e x, g_{a+1}^e g_{a+1}^i g_a^e x)$

$$\begin{aligned}
& \widetilde{V}_{a+1}(g_{a+1}^i g_a^e x, g_{a+1}^e g_{a+1}^i g_a^e x) - \widetilde{V}_{a+1}(g_{a+1}^i x, g_{a+1}^e g_{a+1}^i x) \\
= \; & V_{a+1}(g_{a+1}^i g_a^e x) - V_{a+1}(g_{a+1}^i x)
\end{aligned}$$

Define $\lambda = g_a^e$. It suffices to show that the above is weakly increasing in x. The above is an increasing transformation of the induction hypotheses because λ is independent of $g_{a+1}^i x$. Then the above expression is increasing in x.

2. in K region with $(g_{a+1}^i x, g_{a+1}^e g_{a+1}^i x)$, in S region with $(g_{a+1}^i g_a^e x, g_{a+1}^e g_{a+1}^i g_a^e x)$

$$\begin{aligned}
& \widetilde{V}_{a+1}(g_{a+1}^i g_a^e x, g_{a+1}^e g_{a+1}^i g_a^e x) - \widetilde{V}_{a+1}(g_{a+1}^i x, g_{a+1}^e g_{a+1}^i x) \\
= \; & V_{a+1}(g_{a+1}^e g_{a+1}^i g_a^e x) - \tau - V_{a+1}\left(g_{a+1}^i x\right)
\end{aligned}$$

I want to show that the above in increasing in x. Let $\lambda = g^e_{a+1} g^e_a$. Since λ is independent of $g^i_{a+1}x$ and by the previous induction argument, then the above is weakly increasing in x.

Similarly, it can be shown for the remaining two cases.

3. in E region with $(g^i_{a+1}x, g^e_{a+1}g^i_{a+1}x)$, in E region with $(g^i_{a+1}g^e_a x, g^e_{a+1}g^i_{a+1}g^e_a x)$

4. in S region with $(g^i_{a+1}x, g^e_{a+1}g^i_{a+1}x)$, in S region with $(g^i_{a+1}g^e_a x, g^e_{a+1}g^i_{a+1}g^e_a x)$

This completes the proof.

Proof of Proposition 3:

I want to show that $\widehat{g}_a(x,\tau)$ is weakly increasing in a. Consider assumption G, that states the internal growth of returns is independent of the level. Patent returns evolve over time according to

$$\begin{aligned} g^i_a &= \frac{x'}{z} \\ g^e_a &= \frac{y'}{x'} = \frac{y'}{g^i_a z} \end{aligned}$$

where in the case of assumption G, the joint density function of the random variables g^i_a and g^e, defined as $f_a(g^x, g^y)$ depends upon a but not z.

For convenience of notation, let us rewrite the value function $V(a,x,y)$ as $V_a(x,y)$. To prove the proposition, it suffices to show that the difference $\widetilde{V}^S_a(x,y) - \widetilde{V}^K_a(x,y)$ is decreasing in a, so that $\hat{g}_a(x,\tau)$ is increasing in a.

$$\begin{aligned} & \widetilde{V}^S_a(x,y) - \widetilde{V}^K_a(x,y) \\ = & \ V_a(y) - \tau - V_a(x) \\ = & \ V_a(g^e_a x) - \tau - V_a(x) \\ = & \ (g^e_a - 1)\, x - \tau \\ & + \beta \left[E\left[\widetilde{V}_{a+1}(x', y') | a, g^e_a x \right] - E\left[\widetilde{V}_{a+1}(x', y') | a, x \right] \right] \end{aligned}$$

The argument of the proof is by induction. First, we start in the case of $a = L$ and $L-1$, that is the last and penultimate period of life of a patent

$$\begin{aligned} & \widetilde{V}^S_L(x,y) - \widetilde{V}^K_L(x,y) \\ = & \ x g^e_L - \tau - x \\ = & \ x(g^e_L - 1) - \tau \end{aligned}$$

For $a = L-1$, it is

$$\begin{aligned}
& \widetilde{V}^S_{L-1}(x,y) - \widetilde{V}^K_{L-1}(x,y) \\
= \;& x(g^e_{L-1}-1) - \tau + \beta[E\left[\widetilde{V}_L(x',y')|L-1, g^e_{L-1}x\right] - E\left[\widetilde{V}_L(x',y')|L-1,x\right]]
\end{aligned}$$

There are three cases to study. The first case is the one in which the patent was kept in period $L-1$ as well as in period L. The second case considers the possibility of sale in period $L-1$ and being kept in period L. Finally, the third one analyzes the case that the patent is sold in both periods.

(1) K - K

$$\begin{aligned}
& x(g^e_{L-1}-1) - \tau + \beta[E[\widetilde{V}^K_L(x',y')|L-1, g^y_{L-1}x] - E\left[\widetilde{V}^K_L(x',y')|L-1,x\right] \\
= \;& x(g^e_{L-1}-1) - \tau + \beta[E[V_L(g^i_L g^e_{L-1}x)] - E[V_L(g^i_L x)]] \\
= \;& x(g^e_{L-1}-1) - \tau + \beta\int [v_L(g^i_L g^e_{L-1}x) - v_L(g^i_L x)] f_L(g^i,g^e)dg^i
\end{aligned}$$

Taking as given x, g^e_{L-1}, and g^e_L, $\widetilde{V}^S_a(x,y) - \widetilde{V}^K_a(x,y)$ is decreasing in a because the integral is larger or equal than zero.

Similarly we can also show it for the next two cases.

For a general a,we know that $\left[E\left[\widetilde{V}_a(x',y')|a-1, g^e_{a-1}x\right] - E\left[\widetilde{V}_a(x',y')|a-1,x\right]\right] = 0$ for $a = L$. So, we have to show that

$$\{E\left[\widetilde{V}_{a+1}(x',y')|a, g^e_a x\right] - E\left[\widetilde{V}_{a+1}(x',y')|a,x\right]\} \to 0 \text{ uniformly}$$

Since we know that

$$\begin{aligned}
E\left[\widetilde{V}_a(x',y')|a-1, g^e_{a-1}x\right] &\to 0 \text{ as } a \text{ approaches } L \\
E\left[\widetilde{V}_a(x',y')|a-1, x\right] &\to 0 \text{ as } a \text{ approaches } L
\end{aligned}$$

and by proposition 1 we know that given a and $g^e > 1$ if a trade takes place

$$E\left[\widetilde{V}_a(x',y')|a-1, g^e_{a-1}x\right] \geq E\left[\widetilde{V}_a(x',y')|a-1,x\right]$$

Therefore $\widehat{g}_a(x,\tau)$ increases in a because $f_a(g^i,g^e)$ depends upon a but not on the return of the patent, z.

Finally, I show that the probability of being traded conditional on survival is weakly decreasing as a function of age. Since it is assumed that the process of arrival of offers, $F_{g^e}(u^e)$, is independent of age, and we have already shown that $g_a(x,\tau)$ is weakly increasing in a, then the probability of being traded is weakly decreasing in a.

Proof of Lemma 4:

We want to show that $\left|\frac{\Delta V(a+1,x',y'|z)}{\Delta a}\right| \leq Q$, which rearranging is $|V(a+1,x',y'|z) - V(a,x',y'|z)| \leq Q$. It is sufficient to show that $(V(a,x',y'|z) - V(a+1,x',y'|z)) \leq Q$.

Let M be the maximum value that a return can achieve. It is finite because the distribution of the growth of internal and external returns are bounded above. Then

$$\begin{aligned} V(a,x',y'|z) &= z + \beta E(z') + \beta^2 E(z'') + ... + \beta^{17-a}E(.) \\ &\leq M + \beta M + \beta^2 M + ... + \beta^{17-a}M \\ &= M\frac{(1-\beta^{18-a})}{1-\beta} \end{aligned}$$

Let m be the minimum value that a return can achieve (i.e., considering that the process of internal growth of returns is bounded below by δ). So, we can show that

$$\begin{aligned} V(a+1,x',y'|z) &= z + \beta E(z') + \beta^2 E(z'') + ... + \beta^{17-a-1}E(.) \\ &\leq m + \beta m + \beta^2 m + ... + \beta^{17-a-1}m \\ &= m\frac{(1-\beta^{18-a-1})}{1-\beta} \end{aligned}$$

Therefore, it suffices for Q to be such that $M\frac{(1-\beta^{18-a})}{1-\beta} - m\frac{(1-\beta^{18-a-1})}{1-\beta} \leq Q$.

Proof of Proposition 5:

Given any patent return z_1 and z_2 such that $z_2 > z_1$, I want to show that

$$\frac{\Delta[V(a,x',y'|z_2) - V(a,x',y'|z_1)]}{\Delta a} = \left\{ \begin{array}{ll} > 0 & \text{if} \quad a < a^* \\ \leq 0 & \text{if} \quad a \geq a^* \end{array} \right\}$$

It is sufficient to examine the sign of the relationship between the slopes of the option value of a patent, that is the following expression.

$$\left(\frac{\Delta EV(a+1,x',y'|z_2)}{\Delta a} - \frac{\Delta EV(a+1,x',y'|z_1)}{\Delta a}\right)$$

Without loss of generality, I focus on the case in which the holder of the patent chooses to keep it. Similarly, we can also show the result for the case in which the patent is sold.

$$
\begin{aligned}
& \left(\frac{\Delta EV(a+1,x'|z_2)}{\Delta a} - \frac{\Delta EV(a+1,x',z_1)}{\Delta a}\right) \\
= \quad & \frac{\Delta V(a+1,\delta z_2)}{\Delta a} F_{g_i}(\delta; a, z_2) - \frac{\Delta V(a+1,\delta z_1)}{\Delta a} F_{g_i}(\delta; a, z_1) \\
& + V(a+1,\delta z_2)\frac{\Delta F_{g_i}(\delta; a, z_2)}{\Delta a} - V(a+1,\delta z_1)\frac{\Delta F_{g_i}(\delta; a, z_1)}{\Delta a} \\
& + \int_{\delta^+}^{\infty} [\frac{\Delta V(a+1,u^i z_2|z_2)}{\Delta a} f_{g_i}(u^i; a, z_2) - \frac{\Delta V(a+1,u^i z_1|z_1)}{\Delta a} f_{g_i}(u^i; a, z_1)] \mathrm{d}u^i \\
& + \int_{\delta^+}^{\infty} [V(a+1,u^i z_2|z_2)\frac{\Delta(f_{g_i}(u^i; a, z_2)}{\Delta a} - V(a+1,u^i z_1|z_1)\frac{\Delta(f_{g_i}(u^i; a, z_1)}{\Delta a}] \mathrm{d}u^i
\end{aligned}
$$

where f_{g_i} is the density function of F_{g_i}.

The first term is always negative. To show this, first notice that $\frac{\Delta V(a+1,\delta z_i)}{\Delta a} < 0$ for $i \in \{1,2\}$. It was assumed that $F_{g_i}(\delta; a, z_2) > F_{g_i}(\delta; a, z_1)$. Using the particular case in which F_{g_i} is independent of z we can show that $\frac{\Delta V(a+1,\delta z_2)}{\Delta a} < \frac{\Delta V(a+1,\delta z_1)}{\Delta a}$ always holds. Therefore, the first term is negative.

We can also show that the sign of the second term is positive. By Lemma 1 we know that the value function is increasing in the patent per period returns, so $V(a+1,\delta z_2) > V(a+1,\delta z_1)$. Also we have assumed that the probability of "no learning" (i.e., $g_a^i = \delta$) is increasing in a, that is equivalent to $\frac{\Delta F_{g_i}(\delta;a,z_2)}{\Delta a} > \frac{\Delta F_{g_i}(\delta;a,z_1)}{\Delta a}$. The term is clearly positive.

The sign of the third term is ambiguos. On the one hand, we know that $\frac{\Delta V(a+1,u^i z_2)}{\Delta a} < \frac{\Delta V(a+1,u^i z_1)}{\Delta a}$, however $f_{g_i}(u^i; a, z_2) < f_{g_i}(u^i; a, z_1)$. This result is not surprising. For instance, in the case in which the the process F_{g_i} is independent of z the equivalent of this term would be always negative. Introducing the assumption for which learning is less likely for patents with higher returns makes the expected value of the potential buyer smaller than in the case of independence.

Since we assumed that $\frac{\Delta(f_{g_i}(u^i;a,z_2))}{\Delta a} > \frac{\Delta(f_{g_i}(u^i;a,z_1))}{\Delta a}$, then the sign of the fourth term is positive.

Next, we have to show that for sufficiently small a, the difference between the slopes of the option value for the potential buyer and seller, $\frac{\Delta EV(a+1,x'|z_2)}{\Delta a} - \frac{\Delta EV(a+1,x',z_1)}{\Delta a}$, increases with age. To do so, it is sufficient to show that the second and fourth terms are larger than the first and third term. The strategy is to construct a bound for the term $\frac{\Delta V(a+1,x',y'|z)}{\Delta a}$. By Lemma 2 we know that $\left|\frac{\Delta V(a+1,x',y'|z)}{\Delta a}\right| \leq Q$, where Q is a number sufficiently small and positive. Therefore, this bound permit us to make the first and third term small enough to compare to the other two terms. There exists a a^* such that $a < a^* \leq \overline{a}$ that the second and fourth term dominate the first and the third one. If $a \geq a^*$, then the last three terms become monotonically small converging to zero when learning vanishes at age $a = \overline{a}$ (i.e., the probability of learning is nul) (i.e., this is a particular case of Gibrat's law, see Proposition 2).

Proof of Proposition 6:

The argument of the proof is as follows. It is assumed that the returns due to internal growth of returns are subject to first order stochastically dominance in z. In other words, the higher today's return is, the more likely it is that the return of tomorrow will be high. Now, consider the problem of whether to sell a patent in period L, that is the last period of life of a patent. A patent is sold if $y \geq x + \tau$. Consider now a period before the last, which is $L - 1$. A patent is now sold if the following condition holds

$$y + \beta E(y'|y) \geq x + \beta E(x'|x) + \tau$$

Rearranging this condition, we obtain that

$$y \geq x + \beta[E(x'|x) - E(y'|y)] + \tau$$

However $\beta[E(x'|x) - E(y'|y)] \leq 0$ because of first order stochastically dominance. Then, it must be the case that $\widehat{g}_{L-1} \leq \widehat{g}_L$. The proof can be extended backwards by any number of finite periods. Therefore, $\widehat{g}_a(.)$ is weakly increasing in a.

Proof of Proposition 7:

I want to show that $\widehat{x}_a(\tau)$ is increasing in a. First I show that for any age a there exists a unique $\widehat{x}_a(\tau)$ for which firms are indifferent between keeping and discontinuing a patent. So, $\widehat{x}_a(\tau)$ is defined as $V(a, \widehat{x}_a(\tau)) = 0$. It is assumed that the schedule of renewal fees c_a, when positive, is increasing with age. It is also assumed in the paper that the probability that tomorrow's return is larger than a given number u is weakly decreasing with age. Given that, Lemma 1 demonstrates that the option value function of a patent, $E\widetilde{V}(a+1, x', y'|a, z)$ is weakly decreasing in a. Therefore, the returns that make a firm indifferent between keeping and discontinuing, $\widehat{x}_a(\tau)$, must be increasing as age increases.

Table 8: Summary Statistics of the Unbalanced Panel

Age of Patent (Years)	Active Patents	Not Yet Traded	Already Traded
A. Number of Patents (Small Innovators)			
1	453,683	442,303	11,380
2	417,372	397,124	20,248
3	382,367	355,802	26,565
4	349,013	317,856	31,157
5	258,655	229,154	29,501
6	236,443	205,712	30,731
7	214,895	184,012	30,883
8	194,289	163,847	30,442
9	123,381	101,128	22,253
10	108,948	88,102	20,846
11	94,978	75,795	19,183
12	81,378	64,122	17,256
13	46,095	35,288	10,807
14	36,894	27,968	8,926
15	29,138	21,973	7,165
16	21,512	16,109	5,403
17	14,977	11,110	3,867

Table 9: Summary Statistics of the Unbalanced Panel (All innovators)

Age of Patent (Years)	Active Patents	Not Yet Traded	Already Traded
B. The Size of Owner of a Patent at grant date: Median (and 25 percentile)			
1	51 (5)	51 (5)	17.5 (2)
2	48 (5)	50 (5)	14 (2)
3	49 (5)	52 (5)	15 (3)
4	48 (5)	51 (5)	14 (3)
5	35 (4)	38 (4)	11 (2)
6	37 (4)	41 (4)	11 (2)
7	33 (4)	36 (4)	10 (2)
8	35 (4)	39 (4)	12 (2)
9	35 (4)	39 (4)	11.5 (2)
10	34 (4)	36 (4)	11 (2)
11	32 (3)	36 (4)	10 (2)
12	29 (3)	33 (3)	10 (2)
13	29 (3)	33 (4)	10 (2)
14	24 (3)	27 (3)	9 (2)
15	28 (3)	30 (4)	14 (2)
16	25 (3)	27 (3)	14 (2)
17	28 (3)	30 (4)	20 (3)

Table 10: Summary Statistics of the Unbalanced Panel (All innovators)

Age of Patent (Years)	Active Patents	Not Yet Traded	Already Traded
C. The Number of Patents: (All innovators)			
1	1,586,346	1,559,776	26,570
2	1,443,092	1,395,121	47,971
3	1,309,045	1,245,627	63,418
4	1,179,492	1,105,492	74,000
5	914,169	843,106	71,063
6	831,048	756,591	74,457
7	750,597	675,343	75,254
8	677,442	602,549	74,893
9	451,399	395,089	56,310
10	396,787	343,052	53,735
11	344,790	294,836	49,954
12	295,563	249,988	45,575
13	170,879	141,532	29,347
14	135,886	111,166	24,720
15	108,620	87,977	20,643
16	80,122	64,202	15,920
17	57,034	45,340	11,694