Delay in Strategic Information Aggregation

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Abstract. We construct a model of group decision making in which conflict between the two members in the group prevents them from aggregating any of their private information if the decision has to be voted on without delay. Allowing the two members to vote again when their initial votes disagree facilitates some information aggregation and may improve ex ante welfare. In a model in which the two members vote repeatedly until they agree, in equilibrium they are increasingly more willing to vote their private information after each disagreement. Information is perfectly aggregated within a finite number of rounds. As delay becomes less costly, the two members are less willing to vote their private information, and information aggregation takes longer. Even when the cost of delay becomes arbitrarily small, the ex ante welfare of the two members is higher than when the decision is made without delay for moderate degrees of conflict.

Keywords: repeated voting, conflict resolution, endogenous information.

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1. Introduction

Individuals may disagree with one another on a joint decision because they have conflicting preferences or because they have different private information. Often it is difficult to distinguish between these two types of disagreement because divergent preferences provide incentives for individuals to distort their information. Even though they may share a common interest in some states had the individuals known each other's private information, the strategic distortion of information can still cause disagreement in these states. When disagreements lead to delay in making decisions, it may seem that any decision is better than no decision and costly delay. We argue in this paper, however, that institutionalized delay in the decision making process can serve a useful purpose. In the context of a stylized model of repeated voting, the prospect of costly delay induces the parties to be more forthcoming with their private information. This enhances information aggregation and potentially improves the welfare of the agents relative to the case when the decision has to be made immediately. Even when the delay cost is arbitrarily small, there can be a significant welfare gain.

The constructive role of delay in strategic information aggregation is illustrated in the simplest model that captures the distinction between preference-driven and information-driven disagreements. Although the model is highly stylized, its only essential feature is that there exist states of nature in which agents would disagree when deciding on the basis on their own private information, but would agree if all information were public. Indeed, absent such states, either the agents continue to disagree after sharing their information, in which case there is no room for mutually beneficial information aggregation and delay can only be harmful, or the agents already agree based on their private information, in which case there is no reason to expect delay to occur at all. The particular configuration of preferences and information structures that captures the essential feature can be illustrated with the following example. Imagine that two managers of a corporation, of marketing and R&D divisions, must jointly decide how to enter an emerging market. The marketing division's strategy focuses on pushing existing products through an advertising campaign, and the R&D division's strategy mainly involves developing a new product that targets the emerging market. For some types of the market, one strategy is definitely more effective than the other, in which case the two managers both prefer the more effective strategy, but there are also other types of the market for which the two strategies are equally effective, in which case each manager prefers the strategy of his own division. Suppose that the marketing manager can distinguish the states for which the advertising strategy is more effective from the other states, and similarly the R&D manager can tell the states for which the product development strategy is more effective from the other states. In this environment, a marketing manager who knows that advertising is the more effective strategy has the incentive to convey this information to the R&D manager, who would benefit from such information. The problem is that a marketing manager with the opposite information also has the incentive to mislead the R&D manager into believing otherwise, provided his information indicates that it is sufficiently likely that two strategies are equally effective. When the product development strategy is in fact more effective, we have a situation in which the marketing manager disagrees with the R&D manager based on his own information but would agree with the latter if perfectly informed. This is the kind of environment we study, in which delay can potentially enhance information aggregation and improve welfare. In the above story, if instead the marketing manager can distinguish the states in which the rival strategy is more effective from the other states, and vice versa for the R&D manager, then either the two managers would immediately agree or they would never do. In either case, there is no information aggregation role for delay.

In section 2 we introduce a highly stylized model of collective decision-making to analyze delay in information aggregation. There is a single conflict state in which the two individuals prefer different alternatives, and two equally likely common interest states, one for each alternative, when their preferences coincide. Ex ante, each individual favors a different alternative, and the degree of conflict between the two individuals is captured by the prior probability of the conflict state. In each common interest state, the individual who ex ante favors the mutually preferred alternative is perfectly informed, while the other individual is uninformed and knows only that the state is not the other common interest state. In the conflict state, both individuals are uninformed and each knows only that his ex ante favorite alternative is not mutually preferred. The information structure and preferences are such that if the decision must be made without delay, there is no incentive compatible outcome that Pareto dominates a coin flip between the two alternatives when the degree of conflict is high, even though the state could be a common interest state.

In section 3 we allow the two individuals to vote for a second and final time after paying a delay cost, if they disagree in the first round of voting. There is a generically unique symmetric equilibrium outcome, in which the option of voting again in case of disagreement makes voting by the uninformed types less informative in the first round when the degree of conflict is low, but has the opposite effect when the degree of conflict is high. In the latter case, the softening of the positions taken by the uninformed types improves information aggregation in the first round voting. Moreover, a player may change his vote in the second round upon learning how the other player voted in the first round. The effect of delay cost on the ex ante welfare of the decision makers is non-monotone. Small delay cost does not help resolve disagreement while large delay cost facilitates good decisions at too high a price. There is an intermediate range of delay cost that improves the ex ante welfare of decision makers over a single round of voting.

In many situations collective decisions can only be made with mutual consent. In section 4 we introduce a model of repeated voting until the two sides agree. In the unique symmetric equilibrium of the repeated voting game, the informed type votes his ex ante favorite alternative in every round, while the uninformed type may randomize between the two alternatives. Even though the two players can in principle disagree indefinitely, we find that the information of the two sides is perfectly aggregated within a finite number of rounds. Further, uninformed types make increasingly large "concessions" by voting their ex ante favorite with a smaller probability after each round of disagreement, until either an agreement on the mutually preferred alternative is reached, or the negotiation breaks down because the state is revealed to be the conflict state. A decrease in the delay cost causes the uninformed types to be less willing to make concessions, but increases their equilibrium payoff. By taking the limit as the delay cost goes to zero, we show that the ex ante equilibrium payoff of each individual in the repeated voting game is greater than what they would expect from an immediate coin flip when the degree of conflict is moderate. The constructive role played by delay in improving the quality of information aggregation and welfare is discussed further in section 5, where we also comment on how to extend the current framework to analyze the design of negotiation deadlines, and to study environments with more than two agents such as voting in elections.

The repeated voting model in this paper introduces the novel feature of gradual information aggregation to models of repeated bargaining and negotiation. In a pure bargaining model (Stahl 1972; Rubinstein 1982), the trade off between getting a bigger share of the pie but at a later date helps pin down a unique solution to the bargaining problem which is plagued by multiple equilibria in a one-shot model, even though delay does not occur in equilibrium. There are numerous extensions to the Stahl-Rubinstein model that can generate delay as part of the equilibrium outcome.[1] One strand of this literature relies on asymmetric information about the size of the pie that is being divided. In a model of strikes, for example, a firm knows its own profitability but the firm's unionized workforce does not. Strike or delay is a signaling device in the sense that the willingness to endure a longer work stoppage can credibly signal the firm's low profitability and help it to arrive at a more favorable wage bargain. These bargaining models are private-value problems, as each agent's gains from trade at a given price depend only on his own private information. We instead have a common-value problem: disagreement over the alternatives is not a pure bargaining issue, because individuals in our model would sometimes agree on which is the best alternative had they known the true state. Put differently, voting outcomes in our setup determine the size as well as the division of the pie. We show that delay can play a constructive role in overcoming disagreements that arise from strategic considerations and improving the ex ante welfare of all individuals. Avery and Zemsky (1994) argue that if players are allowed to wait for new information before accepting or rejecting offers, then there is an option value to delay. In our model, no new exogenous information arrives during the voting process. However, the way agents vote provides endogenous information that allows them to update their beliefs and reach better decisions.

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[1] See, for example, Admanti and Perry (1987), Chatterjee and Samuelson (1987), Cho (1990), Cramton (1992), and Kennan and Wilson (1993). There are also bargaining models that generate equilibrium delay through commitment to not accepting offers poorer than past rejected ones (Freshtman and Seidmann 1993; Li 2007), simultaneous offers (Sakovic 1993), multi-lateral negotiations (Cai 2000), and excessive optimism (Yildiz 2004). More closely related to the present paper are recent models of bargaining with interdependent values. See Deneckere and Liang (2006), and Fuchs and Skrzypacz (2008).

Our paper is also related to the literature on debates (Austen-Smith 1990; Austen-Smith and Feddersen 2006; Ottaviani and Sorensen 2001) and voting (Li, Rosen and Suen 2001) in committees. Models of debate typically analyze repeated information transmission as cheap talk, while we emphasize the role of delay cost in multiple rounds of voting.[2] Our setup is the closest to Li, Rosen and Suen (2001). The focus there is on the impossibility of efficient information aggregation. Here, we skirt issues such as quality of private signals and the trade-off between making the two different types of errors, and focus instead on how costly delay can help improve the quality of decisions and increase welfare.

2. The Model

Two players, called Left and Right, have to make a joint choice between two alternatives, l and r. There are three possible states of the world: L, M, and R. We assume that the prior probability of state L and state R is the same, given by $\pi < 1/2$. The relevant payoffs for the two players are summarized in the following table:

	L	M	R
l	$(1,1)$	$(1,1-2\lambda)$	$(1-2\lambda,1-2\lambda)$
r	$(1-2\lambda,1-2\lambda)$	$(1-2\lambda,1)$	$(1,1)$

In each cell of this table, the first entry is the payoff to Left and the second is the payoff to Right. We normalize the payoff from making the preferred decision to 1 and let the payoff from making the less preferred decision be $1-2\lambda$. The parameter $\lambda > 0$ is the loss from making the wrong decision relative to a coin toss. In state L both players prefer l to

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[2] Coughlan (2000) investigates conditions under which jurors vote their signals and aggregate their information efficiently in a model where a mistrial leads to a retrial by a new independent jury. He does not consider the issues of delay that are the focus of the present paper. Farrell (1987) introduces a model in which repeated cheap talk helps players coordinate to arrive at a correlated equilibrium of a battle-of-the-sexes game. There is no issue of efficient information aggregation in that model. More recently, in a dynamic cheap talk model with multiple senders and a receiver who may choose to wait, Eso and Fong (2007) show that when the senders are perfectly informed there is an equilibrium with full revelation with no delay. When the senders are imperfectly informed, Eso and Fong establish conditions under which there exist equilibria converging to full revelation with no delay as the noises in the senders' signals disappear.

r, and in state R both prefer r to l. The two players' preferences are different when the state is M: LEFT prefers l while RIGHT prefers r. In this model there are elements of both common interest and conflict between these two players. Note that LEFT ex ante favors l, while RIGHT's ex ante favorite alternative is r.

The information structure is such that LEFT is able to distinguish whether the state is L or not, while RIGHT is able to distinguish whether the state is R or not. Such information is private and unverifiable. When LEFT knows that the state is L, or when RIGHT knows that the state is R, we say they are "informed;" otherwise, we say they are "uninformed."[3] Without information aggregation, the preference between l and r of an uninformed LEFT depends on the relative likelihood of state M versus state R. Let γ denote his belief that the state is M, given by

$$\gamma = \frac{1-2\pi}{1-\pi}.$$

We note that γ can be interpreted as the ex ante degree of conflict. When γ is high, an uninformed player perceives that his opponent is likely to have different preferences regarding the correct decision to be chosen.

As a useful welfare benchmark, let us consider a single round of simultaneous voting game. Imagine that each player votes l or r simultaneously, with the agreed alternative implemented immediately and any disagreement leading to an immediate fair coin toss between l and r and a payoff of $1-\lambda$ to each player. It is a dominant strategy for an informed player to vote for his ex ante favorite alternative. For uninformed LEFT or RIGHT, the optimal strategy depends on the degree of conflict. If $\gamma < 1/2$, then the dominant strategy for the uninformed players is to vote against their favorite decisions. In states L and R, such equilibrium voting strategies lead to the mutually preferred alternative being chosen, while in state M, the decision is determined by flipping a coin, which is again

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[3] It is not essential for our paper that the informed types are perfectly sure that the state is a common-interest state. The logic of our model remains the same as long as an informed and an uninformed type favor different alternatives on the basis on their private information only, but would recognize a mutually preferred alternative when information is shared. For example, suppose that each player observes a binary signal for or against his ex ante favorite alternative, and prefers his ex ante favorite if and only if there is at least one signal for it. Then, a player who receives a private signal for his ex ante favorite would be similar to an informed type in our setup, while a player who receives a signal against his ex ante favorite would be an uninformed type.

Pareto efficient. In contrast, if $\gamma > 1/2$, then it is a dominant strategy for each uninformed player to vote for his ex ante favorite. The equilibrium outcome is that the two players disagree in every state, and the decision is always determined by flipping a coin, with a payoff of $1 - \lambda$.

The result that information aggregation is impossible for $\gamma > 1/2$ is a robust feature of our configuration of information structure and preferences. Indeed, this configuration is intentionally chosen to yield a stronger result that there is no incentive compatible outcome that Pareto dominates a coin toss when $\gamma > 1/2$. To see this, we apply the revelation principle and consider any direct mechanism that satisfies the incentive compatibility constraints for truthful reporting of private information. Since in a truth-telling equilibrium the true state can be recovered from the reports submitted by the two players, we can write q_R, q_M, and q_L as the probabilities of implementing alternative r when the true states are R, M, and L, respectively. Finally, let $\tilde{q}$ be the probability of implementing r when the reports are inconsistent, that is, when both report that they are informed. The incentive constraints for, respectively, the informed Right, the informed Left, the uninformed Right and the uninformed Left, can be written as:

$$\begin{aligned}
&q_R + (1-q_R)(1-2\lambda) \geq q_M + (1-q_M)(1-2\lambda),\\
&q_L(1-2\lambda) + (1-q_L) \geq q_M(1-2\lambda) + (1-q_M),\\
&\gamma(q_M + (1-q_M)(1-2\lambda)) + (1-\gamma)(q_L(1-2\lambda) + (1-q_L))\\
&\qquad\qquad \geq \gamma(q_R + (1-q_R)(1-2\lambda)) + (1-\gamma)(\tilde{q}(1-2\lambda) + (1-\tilde{q})),\\
&\gamma(q_M(1-2\lambda) + (1-q_M)) + (1-\gamma)(q_R + (1-q_R)(1-2\lambda))\\
&\qquad\qquad \geq \gamma(q_L(1-2\lambda) + (1-q_L)) + (1-\gamma)(\tilde{q} + (1-\tilde{q})(1-2\lambda)).
\end{aligned}$$

The first two incentive constraints imply that $q_R \geq q_M$ and $q_M \geq q_L$; the last two imply that $(1-\gamma)(\tilde{q} - q_L) \geq \gamma(q_R - q_M)$ and $(1-\gamma)(q_R - \tilde{q}) \geq \gamma(q_M - q_L)$, and thus

$$(1-\gamma)(q_R - q_L) \geq \gamma(q_R - q_L).$$

The above is inconsistent with $\gamma > 1/2$ unless $q_R - q_L = 0$. It follows that $q_R = q_M = q_L$ when $\gamma > 1/2$ in any incentive compatible outcome.[4] Since the two players are ex ante

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[4] This result does not depend on the symmetry assumption that the probabilities of the two agreement

symmetric, it is natural to focus on the outcome of $q_R = q_M = q_L = 1/2$, which is equivalent to a fair coin toss. The equilibrium payoff of $1 - \lambda$ in the one-round voting game is a natural welfare benchmark for comparison with the games that allow delay and re-voting in sections 3 and 4 when $\gamma > 1/2$.

The impossibility of information aggregation when $\gamma > 1/2$ assumes that at least one of the two decisions must be taken and that there are no transfers. It is easy to see that efficient information aggregation can be achieved regardless of γ if a sufficiently large monetary penalty can be imposed on both players when their reports are inconsistent. In the ensuing analysis, costly delay plays a similar role of incentive budget-breaking. Although the theoretical underpinning of the constructive role of delay is familiar, in many realistic environments of collective decision making delay is a more natural mechanism than transfers to improve the quality of information aggregation.[5] In section 3, we consider a model in which the two players vote again if their votes in the first round of voting disagree, and if they fail to agree in the second round of voting, a coin flip is used to decide. In section 4, the two players vote repeatedly until they agree. In both models, we assume that the two players incur an additive cost of delay $\delta > 0$ between two adjacent rounds of voting. Such cost may reflect the time and expenses of setting up a second round of meeting and negotiations. An alternative way to model delay cost is to apply a multiplicative discount factor to the payoffs if the decision is implemented in the second round. In this case, delaying a preferred decision is more costly than delaying an inferior decision. Consequently the analysis of the discounting case is slightly more cumbersome than the

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states are the same. No information aggregation is possible if both probabilities are less than the prior probability of state M. In the present model of strategic information aggregation, the signal structure of each player is partitional and binary. This feature is responsible for the result that information aggregation is either ex post efficient, or impossible. In a more general model, ex post inefficiency does not necessarily take the form of impossibility of information aggregation. See Li, Rosen and Suen (2001).

[5] Furthermore, in our model incentive budget-breaking and welfare improvements occur even in the limit of the delay cost becoming arbitrarily small. The importance of incentive budget breaking is well-known. See, for example, Holmstrom's (1982) model of moral hazard in teams, and Myerson and Satterthwaite's (1983) model of bilateral trading with asymmetric information. In the present model of strategic information aggregation, if the two players could commit to a mechanism that imposes an arbitrarily large cost of delay when their reports are inconsistent, then efficient information aggregation would be achieved, with no delay in the truth-telling equilibrium. It is also possible to achieve efficient information aggregation through transfers between the two players instead of costly delay: in the obvious quasi-linear extension of the present model, there is a truth-telling equilibrium with efficient information aggregation if each player is required to make a transfer equal to λ to the other player when his ex ante favorite alternative is chosen.

fixed cost case. We therefore adopt the more transparent assumption of fixed delay cost.[6] The basic insights of this paper do not depend on which of these two assumptions is used.

In the following analysis, we refer to Left voting his ex ante favorite alternative l, and Right voting r, as "persisting," and refer to the opposite as "conceding." In each voting round, there can be two kinds of disagreement: a "regular disagreement," in which the two players both persist; or a "reverse disagreement," in which both concede. When delay cost is large, the inferior alternative can be better than the preferred alternative with delay. In that case, even an informed Right would prefer to vote l if he knows that Left will vote l. The strategic situation is analogous to a "battle-of-the-sexes" game and the main economic issue is that of coordination to one of the two asymmetric outcomes (all voting for l or all voting for r) to avoid the large delay cost. Of course these two outcomes cannot be Pareto ranked. Instead our main concern in this paper is to study whether and how delay can improve the payoff of both players by making information aggregation more efficient. To this effect we focus on equilibria in which the strategies of Right and Left, for both informed and uninformed types, are symmetric to each other, and the informed types always persist in each round. The notion of equilibrium we use in the ensuing analysis is perfect Bayesian equilibrium.

3. Re-voting: Concession, Persistence, and Vote Switching

In this section, we consider a game with possibly two rounds of simultaneous voting. Each individual, Left or Right, can vote for either l or r in each round. If the two votes in the first round agree, then the agreed alternative is implemented immediately and the game ends. If the two votes differ, then each individual has to incur a delay cost δ and the two of them will vote again in the second round. If the two votes agree then the decision is made according to the votes; otherwise, the decision is made by a fair coin toss.

3.1. Equilibrium construction

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[6] The elapsed time between successive voting rounds can be quite short relative to the time for the actual implementation of a decision. In this context, modeling delay as a fixed cost may be more realistic than modeling it as a loss from impatience.

The continuation equilibrium in the second and final round of voting is already described in the previous section. Recall that in this equilibrium the informed type votes for his ex ante favorite. Denote the uninformed type's equilibrium probability of voting for his ex ante favorite alternative by $x^0(\gamma')$, which is given by

$$x^0(\gamma') \begin{cases} = 0 & \text{if } \gamma' \in [0, 1/2), \\ \in [0, 1] & \text{if } \gamma' = 1/2, \\ = 1 & \text{if } \gamma' \in (1/2, 1] \end{cases}$$

where γ' is the belief of the uninformed in the second round. Note that when $\gamma' = 1/2$, there is a continuum of equilibria.[7] Let $U^0(\gamma')$ be the equilibrium payoff to the uninformed types, given by

$$U^0(\gamma') = \begin{cases} 1 - \lambda\gamma' & \text{if } \gamma' \in [0, 1/2), \\ \in [1 - \lambda, 1 - \lambda/2] & \text{if } \gamma' = 1/2, \\ 1 - \lambda & \text{if } \gamma' \in (1/2, 1]. \end{cases} \tag{1}$$

Let $V^0(\gamma')$ be the equilibrium payoff to the informed types, which depends on the belief γ' of the uninformed and is given by

$$V^0(\gamma') = \begin{cases} 1 & \text{if } \gamma' \in [0, 1/2), \\ \in [1 - \lambda, 1] & \text{if } \gamma' = 1/2, \\ 1 - \lambda & \text{if } \gamma' \in (1/2, 1]. \end{cases}$$

Note that $U^0(1) = V^0(1)$ and $U^0(\gamma') \leq V^0(\gamma')$ for all γ'.

Fix a belief γ of an uninformed type in the first round. Consider an equilibrium in which the opposing informed type persists with probability 1, while the opposing uninformed type persists with probability x^1 in the first round. Then, upon a regular disagreement, the uninformed revises his belief that the state is M downward to

$$\gamma' = \frac{\gamma x^1}{\gamma x^1 + 1 - \gamma} \leq \gamma, \tag{2}$$

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[7] If each player pays an additional penalty when the game ends in a disagreement in the final round, the continuum of equilibria at 1/2 is replaced by an interval of beliefs of the uninformed with each belief corresponding to a unique equilibrium probability of persisting by the uninformed. Our results in this paper can be extended to this case without any qualitative change. The continuum of equilibria feature highlights the discontinuity in information aggregation in one-round voting.

unless $\gamma = 1$ and $x_1 = 0$, in which case Bayes' rule does not apply. Upon a reverse disagreement, the uninformed updates his belief that the state is M to 1. Given this strategy profile, the payoff of the uninformed from persisting is:

$$U_p^1(\gamma, x^1) = \gamma(x^1(-\delta + U^0(\gamma')) + (1 - x^1)) + (1 - \gamma)(-\delta + U^0(\gamma')) \tag{3}$$

for all γ and x^1 such that γ' is defined according to (2) and is not equal to 1/2. Let $U_p^1(1, 0) = 1$, and for any $\gamma \geq 1/2$ and $x^1 = (1-\gamma)/\gamma$, let $U_p^1(\gamma, (1-\gamma)/\gamma)$ represent a continuum of payoffs, each corresponding to a value of $U^0(1/2) \in [1 - \lambda, 1 - \lambda/2]$. The payoff to the uninformed type from conceding is

$$U_c^1(\gamma, x^1) = \gamma(x^1(1 - 2\lambda) + (1 - x^1)(-\delta + U^0(1))) + (1 - \gamma) \tag{4}$$

for all $x^1 < 1$. Let $U_c^1(\gamma, 1) = \gamma(1 - 2\lambda) + 1 - \gamma$.

Proposition 1. *In the two-round voting game, there exists a symmetric equilibrium in which in the first round the informed types persist with probability 1 and the uninformed types persist with probability $x^1(\gamma)$, given by*

(i) $x^1(\gamma) = 0,\ \text{if } \gamma \in [0, \delta/(2\delta + \lambda))$,

(ii) $x^1(\gamma) = \min\{1, (\gamma(\delta + \lambda) - (1 - \gamma)\delta)/(2\gamma\delta)\},\ \text{if } \gamma \in [\delta/(2\delta + \lambda), \max\{1/2, 3\delta/(4\delta + \lambda)\})$,

(iii) $x^1(\gamma) = (1 - \gamma)/\gamma,\ \text{if } \gamma \in [\max\{1/2, 3\delta/(4\delta + \lambda)\}, (3\delta + \lambda)/(4\delta + 2\lambda))$,

(iv) $x^1(\gamma) = \min\{1, (2\gamma - 1)(\delta + \lambda)/(2\gamma\delta)\},\ \text{if } \gamma \in [(3\delta + \lambda)/(4\delta + 2\lambda), 1]$.

The proof of Proposition 1 requires comparing $U_p^1(\gamma, x^1)$ and $U_c^1(\gamma, x^1)$ given by equations (3) and (4), and verifying that the informed players have no incentive to deviate from persisting. The details are in the appendix. The equilibrium play characterized by the above proposition is generically unique subject the symmetry restriction and the restriction on the voting behavior of the informed players. To see this, note that for any γ the expected payoff difference $U_p^1(\gamma, x^1) - U_c^1(\gamma, x^1)$ is a continuous correspondence of x^1; except at $x^1 = (1 - \gamma)/\gamma$ for $\gamma \in [1/2, 1)$, the difference is a piecewise linear and strictly decreasing function of x^1; at $x^1 = (1-\gamma)/\gamma$ for $\gamma \in [1/2, 1)$, the difference is multi-valued, each corresponding to a continuation value $U^0(1/2)$ in $[1 - \lambda, 1 - \lambda/2]$. Thus, the equilibrium given in Proposition 1 is unique except when $\gamma = 1/2$ and $\lambda > 2\delta$. In this case, which is contained in case (iii) of the proposition, for the uninformed the expected payoff from persisting is strictly greater than the payoff from conceding when the continuation payoff $U^0(1/2)$ is the highest at $1-\lambda/2$, and thus it is an equilibrium with $x^1 = 1$ and any continuation payoff $U^0(1/2) \geq 1-\lambda+\delta$.[8]

Proposition 1 shows that $x^1(\gamma) \geq x^0(\gamma)$ for any fixed $\gamma < 1/2$, and the opposite is true for $\gamma > 1/2$. Thus, when delay is unlikely because the degree of conflict is low, the possibility of voting again encourages the uninformed to get their own way, while delay is likely with a high degree of conflict so the possibility of voting again makes the uninformed more willing to concede.

The opportunity of voting again for a second time in case of disagreement also means that the uninformed types can learn from the voting outcome in the first round, even though in our model no exogenous new information arrives between the two rounds. We may think of learning in our model as represented by the uninformed types' vote switching between the two rounds. To this end, it is helpful to label the four cases in Proposition 1 according to the first round voting strategy of the uninformed players and how they vote in the second round upon a regular disagreement.

(i) "Conceding equilibrium." The uninformed players concede in both rounds. More precisely, the uninformed type is ready to concede again upon a regular disagreement, which can happen only off the equilibrium path.

(ii) "Switching equilibrium." The uninformed type randomizes in the first round but switches to conceding with probability 1 upon a regular disagreement. If $\max\{1/2, 3\delta/(4\delta+\lambda)\} = 1/2$, or equivalently, if $\lambda > 2\delta$, the uninformed type persists with probability 1 for $\gamma \in [\delta/\lambda, 1/2)$; upon a regular disagreement, the updated belief stays at $\gamma < 1/2$ and the uninformed types switch to conceding in the final round.

(iii) "Random switching equilibrium." The uninformed type randomizes in the first round. Upon a regular disagreement, his updated belief is $\gamma' = 1/2$, so he continues to randomize in the second round.

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[8] In a model where the two players can vote for a maximum of T rounds, there is a continuum of equilibria at $\gamma = 1/2$ if $\lambda > (T+1)\delta$. There is a unique equilibrium otherwise.

(iv) "No switching equilibrium." The uninformed type randomizes in the first round, but never concedes in the next round upon a regular disagreement. When $(\delta + \lambda)/(2\lambda) < 1$, or equivalently, when $\lambda > \delta$, the uninformed type persists with probability 1 for $\gamma \in [(\delta + \lambda)/(2\lambda), 1]$; the updated belief upon a regular disagreement stays at $\gamma > 1/2$ and there is no concession in the final round.

Which type of equilibria obtains depends only on γ and λ/δ. We can partition the parameter space into four regions with the different types of equilibria. See Figure 1. Note that $x^1(\gamma)$ is not monotone in γ due to the discontinuity in the equilibrium play $x^0(\gamma)$ in the second round voting at 1/2. In a switching equilibrium or a no switching equilibrium, $x^1(\gamma)$ is increasing in γ, but in a random switching equilibrium, the value of $x^1(\gamma)$ must be such that the updated belief γ' is fixed at 1/2. In the latter case, the higher the γ, the lower must be $x^1(\gamma)$. In this case, the uninformed players are less tough as the degree of conflict increases. However, in spite of the non-monotonicity of $x^1(\gamma)$, the equilibrium updated belief γ' is increasing in the prior belief γ, with γ' being a constant in a random switching equilibrium, so that a higher degree of conflict in the first round voting in equilibrium results a higher degree of conflict in the second round.

The equilibrium value of $x^1(\gamma)$ is weakly increasing in the error cost λ and weakly decreasing in the delay cost δ (both within each type of equilibrium and across different types). In other words, when the delay cost δ is low, or when the cost of implementing the inferior decision λ is high, the uninformed players will behave in a tougher way by voting more like informed players. When delay cost δ is sufficiently small, the equilibrium outcome in the two-round voting game is a persistent disagreement in both rounds.

3.2. Welfare analysis

Let $U^1(\gamma)$ denote the expected payoff of the uninformed type in the two-round voting game when his belief at the beginning of the first round is given by $\gamma = (1 - 2\pi)/(2\pi)$. Similarly, let $V^1(\gamma)$ be the expected payoff of the informed type. The ex ante equilibrium payoff of each player is

$$W^1(\gamma) = (1 - \pi)U^1(\gamma) + \pi V^1(\gamma) = \frac{1}{2-\gamma}U^1(\gamma) + \frac{1-\gamma}{2-\gamma}V^1(\gamma).$$

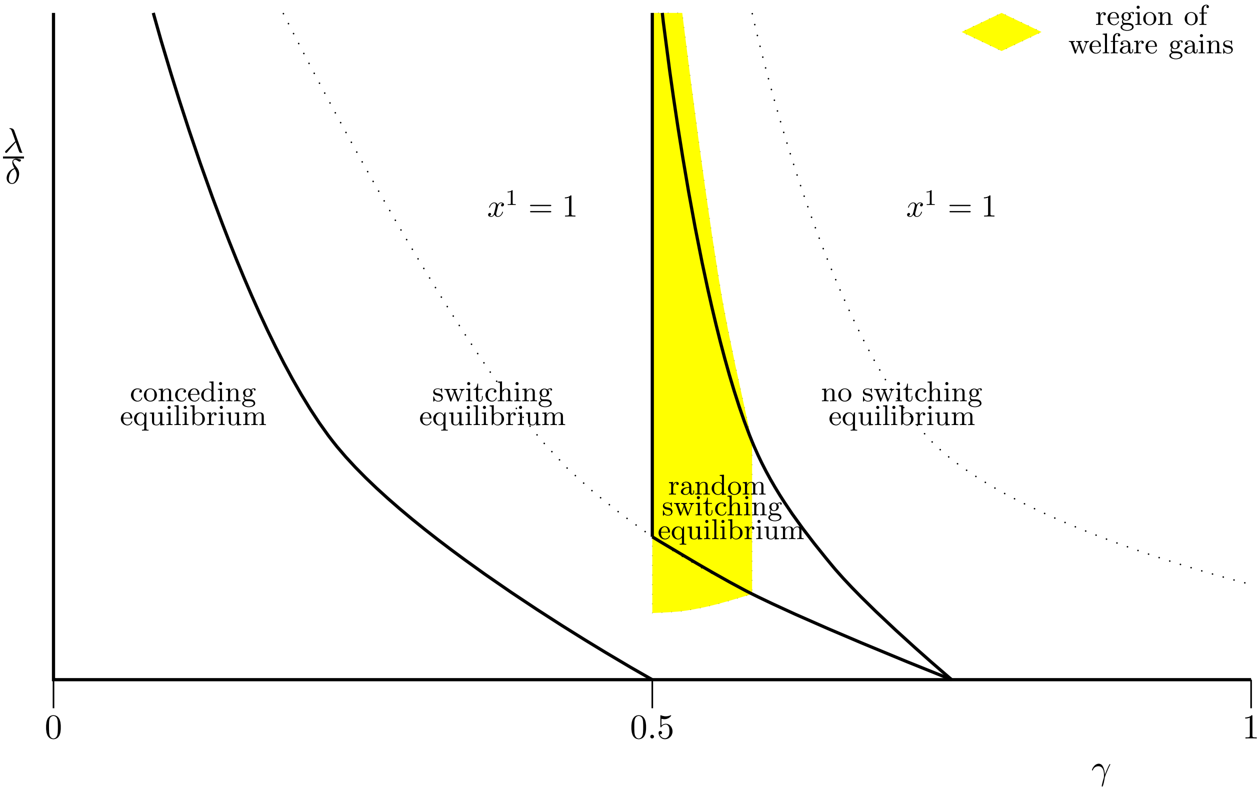

Figure 1

The ex ante equilibrium payoff function W^0 of the one-round voting game analyzed in section 2 can be defined in a similar way as above.

When $\gamma < 1/2$, an uninformed player always yields to his opponent in the one-round voting game, and the resulting equilibrium is Pareto efficient. Since $\delta/(2\delta + \lambda) < 1/2$, adding the possibility of a second round of voting cannot improve the ex ante welfare of the players when the two-round voting game has a conceding equilibrium. Indeed, it is easy to verify that neither an uninformed type nor an informed type can benefit from having an additional round of voting.

When $\gamma > 1/2$, however, the players cannot both do better than flipping a coin with one round of voting. In any equilibrium of the one-round voting game, the probability of implementing the preferred decision cannot exceed 1/2. Allowing an additional round of voting can potentially improve their ex ante welfare as the uninformed player is less tough to his opponent (i.e., $x^1(\gamma) \leq x^0(\gamma)$). As a result, the probability of implementing the preferred alternative in states R and L can be higher than 1/2. To see this, note that in state R, it is always an informed RIGHT (who votes r in both periods) that meets an uninformed LEFT. Any disagreement between the two players in the first round is a regular disagreement, and we can calculate the overall probability of implementing r (in either round). This is given by

$$q^R = \begin{cases} 1 - x^1(\gamma)/2 & \text{in no switching equilibrium,} \\ 1 - x^1(\gamma)x^0(1/2)/2 & \text{in random switching equilibrium,} \\ 1 & \text{in switching equilibrium.} \end{cases}$$

Similarly, the overall probability of implementing r in state L is $q^L = 1 - q^R$. Thus, we have $q^R > 1/2 > q^L$ if $x^1(\gamma) < 1$, so that there is an improvement in information aggregation in these three types of equilibria; indeed, information is aggregated perfectly in a switching equilibrium. From the analysis in section 2 we know that no incentive compatible mechanism without side transfers can implement an outcome with $q^R > q^L$. This result does not apply to our two-round voting game, because we allow the possibility that players have to pay an additional cost δ, which is dissipated as delay rather than transferred to the other player. Thus, it is the possibility of budget breaking that is responsible for potentially improving the equilibrium outcome. Formally, we have the following proposition.

PROPOSITION 2. *For any initial belief of the uninformed* $\gamma \in (1/2, 2 - \sqrt{2})$, *and only for these values of* γ, *there exists an interval of values for* λ/δ *such that* $W^1(\gamma) > W^0(\gamma)$.

The proof of Proposition 2 is in the appendix, where we give the exact lower bound and upper bound on λ/δ as functions of γ such that the ex ante payoff of each player is strictly higher in an equilibrium of the two-round game than in the equilibrium of the one-round game. The region of welfare improvement is shown in Figure 1. Intuitively, welfare gains from introducing the possibility of an additional round of voting exist only when the degree of conflict is not too high. In particular, when γ is close to 1 so that there is little room for a first-round concession or subsequent voting switching, the impact of allowing another round of voting is that the players incur the cost of delaying the eventual disagreement. More generally, Proposition 2 shows that when the degree of conflict is sufficiently high ($\gamma > 2-\sqrt{2}$), any improvement in the quality of the decision is outweighed by the direct cost of delay. Moreover, it can be seen from the proof that this region of welfare improvement contains a subset of each region in the parameter space corresponding to a no switching equilibrium, a random switching equilibrium, and a switching equilibrium. In other words, for each of these three types of equilibria, there are values of γ and λ/δ such that delay increases the ex ante welfare of the players.

The cost of delay has a non-monotone effect on players' welfare. Fix γ at some value between $1/2$ and $2-\sqrt{2}$. When δ is sufficiently low, the equilibrium is a no switching equilibrium with $x^1(\gamma)=1$. Since there is complete disagreement between the players in both rounds, the only effect of introducing re-voting is that players have to pay the delay cost. As δ increases, $x^1(\gamma)$ eventually becomes strictly less than 1. A lower $x^1(\gamma)$ is beneficial to the players when the state is L or R because it increases the chance of arriving at the preferred decision in these states. When δ is sufficiently large, the equilibrium is a switching equilibrium and the probability of reaching the preferred decision in states L and R is equal to 1. However, the probability of incurring the delay cost, given by

$$(1-\pi)x^1(\gamma)+\pi((x^1(\gamma))^2+(1-x^1(\gamma))^2),$$

is increasing in δ as $x^1(\gamma)$ becomes very small, so that the total delay cost necessarily increases. Beyond a certain point, the direct cost of delay outweighs the benefits from improving the quality of decisions, and the two-round voting game yields lower ex ante welfare than the benchmark one round voting game. Nevertheless, Proposition 2 shows that there is an intermediate range of δ such that introducing the possibility of delay will strictly improve the welfare of the players.

4. Repeated Voting: Increasing Concessions and Negotiation Breakdowns

In this section, we consider a model in which the two players vote until they agree, so that there is no final round. We will first construct one such equilibrium and then argue that it is unique. Due to the absence of the final round, the constructed equilibrium is more intuitive than the one characterized in Proposition 1. In particular, the discontinuity in the equilibrium play in the final round no longer has any impact, and the equilibrium strategy of the uninformed is monotone in the belief. The repeated voting game in this section also offers more straightforward comparative statics results on welfare.

4.1. Equilibrium construction and characterization

Since the game is symmetric and since the uninformed types vote for their ex ante favorites with the same probability on the equilibrium path, they have the same belief about the state being the conflict state after any observed sequence of disagreeing votes. For the equilibrium constructed below, it is sufficient to consider equilibrium play when the uninformed types hold the same beliefs. For each such common belief $\gamma \in [0,1]$ that the uninformed types hold regarding the conflict state M, we denote by $x(\gamma) \in [0,1]$ the equilibrium probability that the uninformed types persist. Let $U(\gamma)$ and $V(\gamma)$ be the equilibrium expected payoffs of the uninformed and informed types respectively.[9]

To construct an equilibrium, first we identify an equilibrium play when the uninformed players believe that the state is M with probability 1, in which they persist with probability $x(1)$. It follows from the indifference condition for the uninformed types that

$$U(1) = x(1)(-\delta + U(1)) + (1 - x(1)) = x(1)(1 - 2\lambda) + (1 - x(1))(-\delta + U(1)).$$

Solving these two equations gives a unique pair of equilibrium values

$$U(1) = 1 - \lambda - \sqrt{\delta^2 + \lambda^2}; \quad x(1) = \frac{-\delta + \lambda + \sqrt{\delta^2 + \lambda^2}}{2\lambda}. \tag{5}$$

We note that $x(1) \in (1/2, 1)$ and $U(1) < 1 - 2\lambda$.

Next, we identify an equilibrium play when $\gamma = 0$. Since the uninformed Right believes that the state is L and his opponent (who is informed) votes l, voting l to obtain the preferred decision is strictly better than voting r. Thus, we have $x(0) = 0$ and $U(0) = 1$. Given this, we claim that it is an equilibrium when γ is positive but sufficiently small for the uninformed types to concede with probability 1. To see this, note that $x(\gamma) = 0$ implies

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[9] We have implicitly restricted to stationary strategies that depend only on the belief. There are no non-stationary symmetric equilibria in which the informed types always persist. In particular, it cannot be an equilibrium in which the uninformed types coordinate to persist in some given time period, followed by randomizing between persisting and conceding. This follows because each uninformed type would have unilateral incentive to concede in the time period when both uninformed types are supposed to persist.

that the updated belief upon a regular disagreement is $\gamma' = 0$. Therefore, the payoff to the uninformed Right from voting r is $\gamma + (1-\gamma)(-\delta + U(0))$, and his payoff from voting l is $\gamma(-\delta + U(1)) + (1-\gamma)$. Conceding is strictly preferred to persisting if and only if

$$\gamma < \frac{\delta}{(1+\delta - U(1)) + \delta} \equiv G_1. \tag{6}$$

The corresponding equilibrium payoff of the uninformed types takes the linear form of

$$U(\gamma) = 1 - (1+\delta - U(1))\gamma. \tag{7}$$

We refer to the interval $[0, G_1]$ as the "conceding region."

For γ just above G_1, we construct an equilibrium in which $x(\gamma)$ is such that the one-step updated belief γ' falls into the conceding region. We will then try to identify a one-step interval $[G_1, G_2]$, and so on. That is, there exists an infinite sequence, $G_0 < G_1 < G_2 < \ldots$, with $G_0 = 0$ and $\lim_{k\to\infty} G_k = 1$, such that if $\gamma \in (G_k, G_{k+1}]$ for $k = 1, 2, \ldots$, then $x(\gamma) \in (0,1)$ is such that the updated belief after a regular disagreement satisfies

$$\gamma' = \frac{\gamma x(\gamma)}{\gamma x(\gamma) + 1 - \gamma} \in (G_{k-1}, G_k].$$

Furthermore, the payoff function for the uninformed is piecewise linear of the form

$$U(\gamma) = a_k - b_k \gamma \tag{8}$$

for $\gamma \in (G_k, G_{k+1}]$, with $a_0 = 1$ and $b_0 = 1 + \delta - U(1)$ from equation (7). We construct the sequences of $\{G_k\}$ and $\{(a_k, b_k)\}$ recursively, starting from G_1 and (a_0, b_0).

Fix any $\gamma \in (G_k, G_{k+1}]$ for $k \geq 1$. Assuming that the continuation payoff is given by equation (8), the expected payoff to the uninformed from persisting is

$$(\gamma x + 1 - \gamma)(-\delta + a_{k-1} - b_{k-1}\gamma') + \gamma(1-x) = (\gamma x + 1 - \gamma)(-\delta + a_{k-1}) - \gamma x b_{k-1} + \gamma(1-x).$$

The payoff from conceding is

$$\gamma(x(1-2\lambda) + (1-x)(-\delta + U(1))) + (1-\gamma).$$

The uninformed is indifferent between persisting and conceding when x is given by

$$x(\gamma) = \frac{\gamma b_0 - (1-\gamma)(1+\delta - a_{k-1})}{\gamma(b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda)}. \tag{9}$$

Using Bayes' rule

$$\frac{G_{k+1} x(G_{k+1})}{G_{k+1} x(G_{k+1}) + 1 - G_{k+1}} = G_k,$$

with $x(G_{k+1})$ given in equation (9), we can define G_{k+1} as follows:

$$G_{k+1} = \frac{1 + \delta - a_{k-1} + G_k(b_0 + b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)}. \tag{10}$$

Note that $x(\gamma)$ is increasing in γ from equation (9), implying that the updated belief γ' after a regular disagreement falls in the interval $(G_{k-1}, G_k]$. Finally, substituting equation (9) into the expression for the payoff from voting r, we can verify that $U(\gamma)$ is indeed piece-wise linear of the form given in equation (8), where

$$a_k = 1 - \frac{(1+\delta - a_{k-1})(b_0 - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda}; \quad b_k = 2\lambda + \frac{(b_0 - 2\lambda)(b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + b_{k-1} - 2\lambda}. \tag{11}$$

The above is a pair of difference equations for the sequence $\{(a_k, b_k)\}$. We have the following preliminary results regarding the sequences $\{G_k\}$ and $\{(a_k, b_k)\}$. The proof is in the appendix.

LEMMA 1. *(i)* $a_k \leq 1$ *and* $b_k > 2\lambda$ *for all* k*; (ii) both* a_k *and* b_k *are decreasing in* k*; (iii)* $\lim_{k\to\infty} a_k$ *exists and is given by* $a_\infty = 1 + \lambda - \sqrt{\delta^2 + \lambda^2}$*, and* $\lim_{k\to\infty} b_k$ *exists and is* $b_\infty = 2\lambda$*; (iv)* $0 < G_k < G_{k+1} < 1$ *for all* $k \geq 1$*; and (v)* $\lim_{k\to\infty} G_k = 1$.

The above piece-wise construction of $x(\gamma)$ and $U(\gamma)$ ensures that the strategy of the uninformed types is consistent with equilibrium. It remains to verify that the informed types have no incentive to deviate by voting against their ex ante favorite alternatives. This is established below by showing that given the equilibrium strategy of the uninformed types, the informed types have stronger incentives than the uninformed types to vote for their ex ante favorite alternative.

PROPOSITION 3. *There exists an equilibrium of the repeated voting game in which the strategy of the uninformed types is given by* $x(\gamma)$ *and their payoff is given by* $U(\gamma)$*.*

Proof. First, for $\gamma = 1$, since his opponent is persisting with probability $x(1)$, the informed type is indifferent between persisting and conceding, and his equilibrium payoff is $V(1) = U(1)$.

Next, for $\gamma \in [G_0, G_1]$, since his opponent is conceding with probability 1, the payoff for the informed type from persisting is 1, while his payoff from conceding is $-\delta + V(1) < 1$, implying $V(\gamma) = 1 \geq U(\gamma)$, with equality only if $\gamma = 0$.

Finally, for $\gamma \in (G_1, 1)$, we first establish by induction that $V(\gamma) > U(\gamma)$ for all $\gamma < 1$, as follows. Consider any $\gamma \in [G_k, G_{k+1}]$ and $k \geq 1$, with $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma) \in (G_{k-1}, G_k]$. We obtain

$$\begin{aligned} V(\gamma) &> (\gamma x(\gamma) + 1 - \gamma)(-\delta + V(\gamma')) + \gamma(1 - x(\gamma)) \\ &> (\gamma x(\gamma) + 1 - \gamma)(-\delta + U(\gamma')) + \gamma(1 - x(\gamma)) = U(\gamma), \end{aligned}$$

where the first inequality follows from the fact that $x(\gamma) < \gamma x(\gamma) + 1 - \gamma$, the second inequality follows from the induction hypothesis, and the last equality follows because the uninformed type is indifferent between persisting and conceding for $\gamma \in [G_k, G_{k+1}]$ for $k \geq 1$. Moreover, from the indifferent condition of the uninformed type we obtain

$$\gamma\left(x(\gamma)(-\delta + U(\gamma') - 1 + 2\lambda) + (1 - x(\gamma))(1 + \delta - U(1))\right) + (1 - \gamma)(-\delta + U(\gamma') - 1) = 0.$$

Note that the last term is strictly negative, and so the expression in the square bracket is strictly positive. Since $V(1) = U(1)$, and $V(\gamma') > U(\gamma')$, this implies that

$$x(\gamma)(-\delta + V(\gamma')) + 1 - x(\gamma) > x(\gamma)(1 - 2\lambda) + (1 - x(\gamma))(-\delta + V(1)).$$

The left-hand-side of the above inequality is the equilibrium payoff for the informed type from persisting. The right-hand-side is the deviation payoff from conceding, because after a reverse disagreement the uninformed type is convinced that the state is M. Thus, the informed type strictly prefers persisting to conceding. *Q.E.D.*

The equilibrium represented by equations (9) and (8) is continuous and monotone with respect to the degree of conflict γ. Note that the continuity of $x(\gamma)$ in γ is not required for the construction to be an equilibrium. Nor it is automatic from the construction, because the equilibrium strategy to the left and inclusive of $\gamma = G_k$ is constructed in the interval $(G_{k-1}, G_k]$ while $x(\gamma)$ just to the right of G_k is separately constructed in the next step of $[G_k, G_{k+1})$. The continuity and monotonicity of $x(\gamma)$ is indirectly established below by showing that the payoff function U is continuous. See the appendix.

PROPOSITION 4. *The equilibrium strategy* $x(\gamma)$ *in the repeated voting game is continuous and increasing for all* $\gamma \in [0, 1]$.

The monotonicity result of Proposition 4 provides an intuitive description of the equilibrium behavior. In each round of voting, there are four possible outcomes: an immediate agreement on r, an immediate agreement on l, a regular disagreement, or a reverse disagreement. We interpret a reverse disagreement as a breakdown of the negotiation process. Once a reverse disagreement occurs, it is revealed that what is a good decision for one player is necessarily an inferior decision for the other player. The continuation game is a version of war of attrition game, where each uninformed player chooses the stationary strategy represented by $x(1)$ until they reach a decision.[10] Upon a regular disagreement, on the other hand, the uninformed player becomes more convinced that he is playing against an informed type. The informed type continues to vote for his favorite alternative, but the uninformed player will "soften" his position as $x(\gamma') < x(\gamma)$. In a sense, the negotiation between the two players is making progress, because the probability of choosing the mutually preferred alternative rises if the state is L or R. Moreover, for any γ not arbitrarily close to 1, it only takes a finite number of rounds of regular disagreement before the uninformed player yields to his opponent completely by switching to voting against his ex ante favorite (i.e., $x(\gamma) = 0$), provided there is no breakdown of negotiation before that. Once the game reaches this conceding region, there is either an agreement on the mutually preferred alternative, or the negotiation breaks down and the two uninformed players engage in a war of attrition by adopting the strategy of voting for his ex ante favorite alternative with probability $x(1)$.

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[10] In our version of the war of attrition game, "stopping" corresponds to voting against one's ex ante favorite alternative. Unlike the standard war of attrition game, when both players vote against their favorite, we have a reverse disagreement and the game continues.

The equilibrium in Proposition 3 is unique. That is, if there is a function $y(\gamma)$ defined on $\gamma \in [0,1]$ such that it is an equilibrium for the uninformed types with belief γ to persist with probability $y(\gamma)$, then $y(\gamma) = x(\gamma)$ for all $\gamma \in [0,1]$. To see this, first note that the argument leading to (5) establishes that $y(1) = x(1)$ in any equilibrium. Second, we argue that in any equilibrium $y(\gamma) = 0$ for any $\gamma \in [0, G_1]$. This follows because regardless of the continuation plays, when γ is sufficiently small, the payoff to the uninformed from voting persisting is strictly lower than the payoff from voting conceding regardless of the strategy of the opposing uninformed type. Third, for any $\gamma > G_1$, if the equilibrium $y(\gamma)$ is such that there is a unique continuation value $U(\gamma')$ as given by Proposition 3, then $y(\gamma)$ equals $x(\gamma)$ because the latter is the only value that simultaneously satisfies the equilibrium indifference condition of the uninformed types and Bayes' rule. Finally, because $y(\gamma) = 1$ is never part of equilibrium strategy, in any equilibrium the uninformed types are indifferent between conceding and persisting whenever their belief is strictly higher than G_1. It follows that in any equilibrium $y(\gamma)$ is bounded away from 1. Then the second claim and third claim above imply that $y(\gamma) = x(\gamma)$ for all γ.

4.2 Equilibrium welfare and comparative statics

To analyze the welfare properties of the equilibrium constructed in the repeated voting game, we first derive the payoff function of the informed types. Recall that for any belief γ of the uninformed, $V(\gamma)$ is the equilibrium expected payoff of the informed types. Given the equilibrium strategy $x(\gamma)$ of the uninformed, $V(\gamma)$ satisfies the following recursive formula:

$$V(\gamma) = x(\gamma)(V(\gamma') - \delta) + 1 - x(\gamma), \tag{12}$$

where $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma)$ is the updated belief of the uninformed after a regular disagreement. Using the characterization of $x(\gamma)$ in Proposition 3, we have the following result about V.

Lemma 2. *There exists a sequence $\{(c_k, d_k)\}$, with $c_k \leq 1$ decreasing in k and $\lim_{k\to\infty} c_k = U(1)$, and $d_k \geq 0$ increasing in k and $\lim_{k\to\infty} d_k(1-G_k)/G_k = 0$, such that*

$$V(\gamma) = c_k + d_k \frac{1-\gamma}{\gamma} \tag{13}$$

for any $\gamma \in (G_k, G_{k+1}]$, $k \geq 1$.

The proof of the lemma is in the appendix, where we establish the following system of difference equations for $\{(c_k, d_k)\}$:

$$c_k = 1 - \frac{b_0(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}; \; d_k = d_{k-1} + \frac{(1+\delta-a_{k-1})(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}. \quad (14)$$

By the proof of Proposition 4, the payoff function $V(\gamma)$ is continuous for all $\gamma \in [0,1]$, as is $U(\gamma)$. However, while $U(\gamma)$ is decreasing and piece-wise linear in γ, and is convex because b_k decreases with k, the payoff function $V(\gamma)$ is piece-wise convex but since d_k is increasing in k, at each kink G_k, the left derivative is smaller than the right derivative. Further, from the proof of Proposition 3 we know that the two payoff functions satisfy $V(\gamma) \geq U(\gamma)$ for all $\gamma \in [0,1]$, with equality only at $\gamma = 0$ and $\gamma = 1$.[11]

The equilibrium welfare of the informed and uninformed types depend on the delay cost δ. In the following proposition, we establish that as δ decreases, the conceding region becomes smaller; further, the equilibrium voting by the uninformed types becomes tougher for any degree of conflict. Correspondingly, for any initial degree of conflict, as δ decreases, it takes a greater number of regular disagreements to reach the conceding region. However, in spite of the tougher positions taken by the uninformed types, their equilibrium expected payoffs increase unambiguously because the direct impact of a lower cost of delay per-round dominates. The proof is in the appendix.

PROPOSITION 5. *In the symmetric equilibrium of the repeated voting game, as* δ *decreases,* G_k *strictly decreases for each* $k \geq 1$, $x(\gamma)$ *strictly increases for all* $\gamma \in (G_1, 1]$, *and* $U(\gamma)$ *strictly increases for all* $\gamma \in (0,1]$.

For the informed types, the effect of a decrease in the delay cost δ turns out to be generally ambiguous. The uninformed types toughen their positions, which means longer delays before the mutually preferred alternative is chosen, but each round of disagreement is less costly. The difference equations (11) and (14) allow an explicit calculation of the payoff functions of the informed and the uninformed. In Figure 2, we plot these functions

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[11] While the limit of d_k as k goes to infinity does not exist, the product $d_k(1-\gamma)/\gamma$ converges to 0 because γ goes to 1 as k grows arbitrarily large, which is why $V(1) = U(1)$.

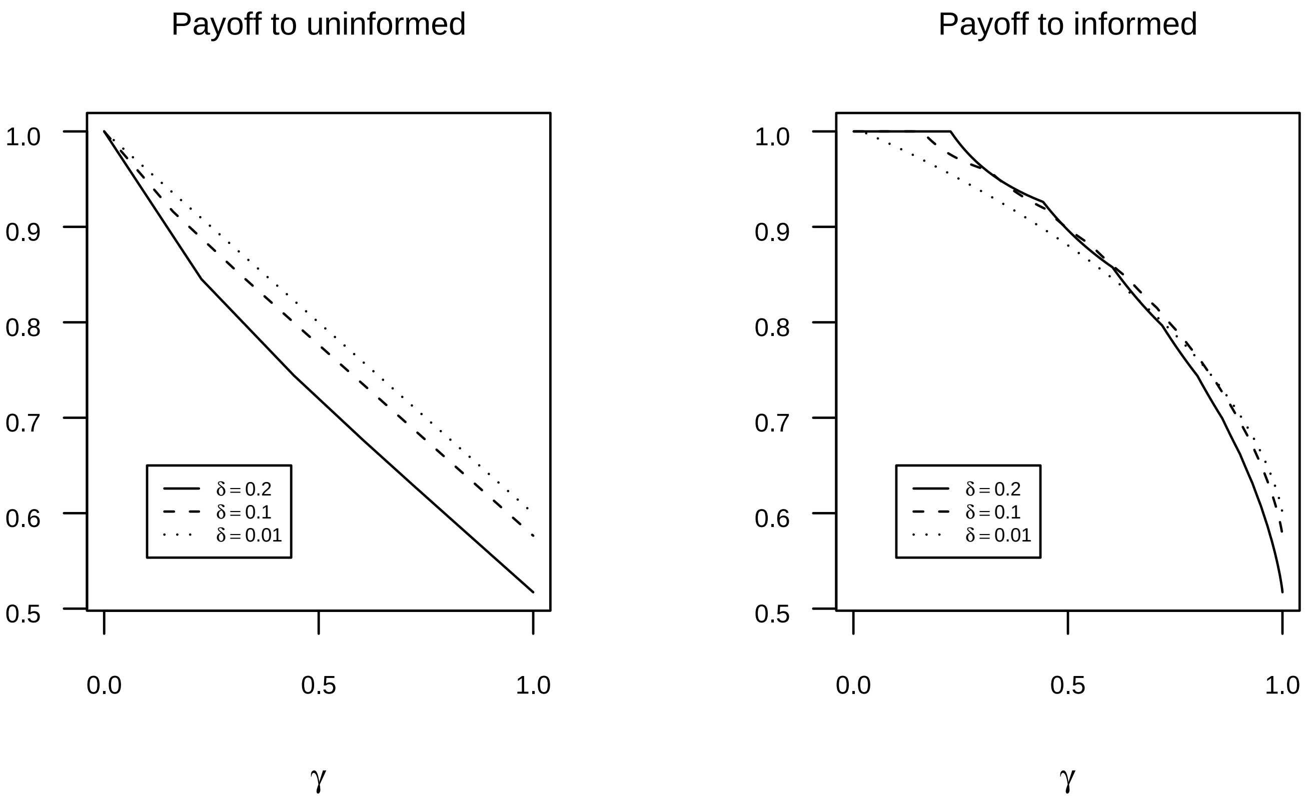

Figure 2

for $\delta = 0.2$, $\delta = 0.1$, and $\delta = 0.01$ (holding λ fixed at 0.2). Consistent with Proposition 5, we see that $U(\gamma)$ is decreasing in δ for any γ. However, $V(\gamma)$ is not monotone in δ.

We now compare the equilibrium payoffs of the uninformed and informed types with their corresponding benchmarks when there is no possibility of delay. For the uninformed, it is immediate that there is no possibility of welfare gain relative to the benchmark of one-round voting regardless of the degree of conflict γ or the delay cost δ. This follows from (1), and because $U(\gamma)$ is a decreasing function, with a slope b_k that is strictly larger than 2λ by Lemma 1, implying that

$$U(\gamma) \leq 1 - 2\lambda\gamma \leq U^0(\gamma).$$

On the other hand, Figure 2 shows that welfare gains are possible for the informed types. In this figure, $V(1/2)$ is greater than $1 - \lambda = 0.8$ for the various values of δ indicated.

The ex ante payoff function before the realization of informed or uninformed types is given by

$$W(\gamma) = \frac{1}{2-\gamma}U(\gamma) + \frac{1-\gamma}{2-\gamma}V(\gamma).$$

A simple characterization of the condition for $W(\gamma)$ to exceed the benchmark welfare $W^0(\gamma)$ in one-round voting is not easy because the payoff functions $U(\gamma)$ and $V(\gamma)$ have infinitely many kink points at $G_1, G_2, \ldots$. However, Figure 2 suggests that U and V approach smoothly differentiable functions as δ becomes small. We therefore undertake to study the welfare comparison through the limiting case of arbitrarily small delay cost. The limit functions $\lim_{\delta\to 0} U(\gamma)$ and $\lim_{\delta\to 0} V(\gamma)$ are well-defined because the equilibrium given in Proposition 3 is continuous in δ.

Proposition 6. *There exists a threshold value $\overline{\gamma} \in (1/2, 1)$ such that for any initial belief $\gamma \in (1/2, \overline{\gamma})$, $W(\gamma) > W^0(\gamma)$ when the delay cost δ is sufficiently small.*

Proof. From equation (5), we obtain $\lim_{\delta\to 0} U(1) = 1 - 2\lambda$, implying that $\lim_{\delta\to 0} b_0 = 2\lambda$. It follows from the difference equations (11) that $\lim_{\delta\to 0} a_k = 1$ and $\lim_{\delta\to 0} b_k = 2\lambda$ for any k. It is then straightforward to show from (6) that $\lim_{\delta\to 0} G_1 = 0$, and from (10) by induction that, for any $k \geq 1$,

$$\lim_{\delta\to 0}(G_{k+1} - G_k) = 0.$$

For any $\delta > 0$ and $\gamma \in (0,1)$, let $\kappa(\delta;\gamma)$ represent the smallest integer k such that $G_k \geq \gamma$. Since the U function is decreasing, the value of $U(\gamma)$ is bounded above by $a_{\kappa(\delta;\gamma)} - b_{\kappa(\delta;\gamma)} G_{\kappa(\delta;\gamma)-1}$, and is bounded below by $a_{\kappa(\delta;\gamma)} - b_{\kappa(\delta;\gamma)} G_{\kappa(\delta;\gamma)}$. Since $\lim_{\delta\to 0} a_{\kappa(\delta;\gamma)} = 1$ and $\lim_{\delta\to 0} b_{\kappa(\delta;\gamma)} = 2\lambda$, and since $\lim_{\delta\to 0} G_{\kappa(\delta;\gamma)} = \lim_{\delta\to 0} G_{\kappa(\delta;\gamma)-1} = \gamma$, the upper bound and the lower bound converge to the same limiting value of

$$\lim_{\delta\to 0} U(\gamma) = 1 - 2\lambda\gamma.$$

For the informed types, we note from the difference equation (14) for $\{c_k\}$ and the limit values of a_k and b_k that $\lim_{\delta\to 0} c_k = 1 - 2\lambda$ for any k. To calculate the limit value of $d_{\kappa(\delta;\gamma)}$, we write

$$d_{\kappa(\delta;\gamma)} = d_1 + \sum_{k=2}^{\kappa(\delta;\gamma)} \frac{d_k - k_{k-1}}{G_k - G_{k-1}}(G_k - G_{k-1}).$$

Letting $v_k = 1 + \delta - a_k$ and $s_k = b_k - 2\lambda$, and using the difference equations (14) and (10), this can be expresses as:

$$d_{\kappa(\delta;\gamma)} = d_1 + \sum_{k=2}^{\kappa(\delta;\gamma)} \frac{1 + \delta - c_{k-1}}{1 - G_{k-1}} \frac{b_0 + v_{k-2} + G_{k-1}s_{k-2}}{b_0 + v_{k-1} + s_{k-1}} \frac{v_{k-1}}{v_{k-2} + G_{k-1}s_{k-2}}(G_k - G_{k-1}).$$

As δ goes to 0, the first term in the summand goes to $2\lambda/(1-G_{k-1})$, the second term goes to 1, and the third term goes to 1 (for any $k \geq 2$, the limit of v_{k-2}/v_{k-1} is 1 and the limit of s_{k-2}/v_{k-1} is 0 as δ goes to 0). Moreover, d_1 goes to 0 as δ goes to 0. Therefore

$$\lim_{\delta \to 0} d_{\kappa(\delta;\gamma)} = \lim_{\delta \to 0} \sum_{k=2}^{\kappa(\delta;\gamma)} \frac{2\lambda}{1-G_{k-1}}(G_k - G_{k-1}) = \int_0^\gamma \frac{2\lambda}{1-G} \mathrm{d}G = -2\lambda \ln(1-\gamma),$$

where the second equality uses the definition of the Riemann integral. Since $V(\gamma)$ is bounded above by $V(G_{\kappa(\delta;\gamma)-1})$ and is bounded below by $V(G_{\kappa(\delta;\gamma)})$, and these two bounds converge to the same limiting value as δ goes to 0, we have

$$\lim_{\delta \to 0} V(\gamma) = 1 - 2\lambda - 2\lambda \ln(1-\gamma)\frac{1-\gamma}{\gamma}.$$

It follows immediately from the limiting payoff functions that

$$\lim_{\delta \to 0} W(1/2) = \frac{2}{3}(1-\lambda) + \frac{1}{3}(1 - 2\lambda(1-\ln 2)) > 1 - \lambda.$$

Moreover, the limit of $\lim_{\delta \to 0} W(\gamma)$ as γ goes to 1 is $1 - 2\lambda$, which is strictly less than $1 - \lambda$. Since the limit function $\lim_{\delta \to 0} W(\gamma)$ is decreasing in γ, there exists a $\overline{\gamma} \in (1/2, 1)$ such that $\lim_{\delta \to 0} W(\gamma) > W^0(\gamma)$ if and only if $\gamma \in (1/2, \overline{\gamma})$. The proposition follows by continuity of the ex ante welfare function in the delay cost δ. *Q.E.D.*

Proposition 5 establishes that $U(\gamma)$ increases as δ decreases. When δ goes to 0, $U(1/2)$ approaches a limiting value of $1 - \lambda$, implying that the uninformed types get the same payoff in equilibrium as the benchmark payoff when $\gamma = 1/2$. We have shown in the proof of Proposition 3 that the payoff to the informed is strictly higher than the payoff to the uninformed for any $\gamma \in (0, 1)$. This already suggests that the informed types will be better off in the repeated voting game than in the benchmark one-round voting game when $\gamma = 1/2$ and δ is small. Proposition 6 makes this precise by deriving an explicit characterization of the limit functions $\lim_{\delta \to 0} U(\gamma)$ and $\lim_{\delta \to 0} V(\gamma)$, ensuring that $V(\gamma) - U(\gamma)$ does not go to 0 as δ goes to 0.

In this repeated voting game, as in the case of the two-round voting game, ex ante welfare gains relative to the benchmark one-round voting game exist only for values of γ

close enough to and greater than 1/2. If the the degree of conflict γ is too large, since $V(1) = U(1) < 1 - \lambda$, the continuity of V implies that $V(\gamma)$ is smaller than the benchmark expected payoff of the informed types for γ close to 1. Also, as in the case of the two-round voting game, the delay cost δ cannot be too great for ex ante welfare gains to exist. From equations (6) and (10) we can verify that for δ sufficiently great, $G_1 < 1/2 < G_2$, and then from (9) we can verify that $x(1/2)$ is bounded away from 0 for sufficiently great δ, implying that $V(1/2)$ falls below the benchmark payoff of $1 - \lambda$ if δ is sufficiently great.

5. Discussion

The budget-breaking role of costly delay is robust to the game form in the repeated voting game. Imagine a repeated voting game with costly delay which differs from our game only in that after a reverse disagreement the game ends with an immediate coin toss. Analysis for this game follows in a parallel fashion as what we have done in our paper. It turns out that in the limit of the delay cost converging to 0, this new game has the same equilibrium outcome as our game. The same is true for any game defined by replacing the equilibrium continuation payoff after a reverse disagreement with any feasible continuation payoff. In particular, in the equilibrium constructed in Proposition 3, the continuation payoff to the uninformed after a reverse disagreement is $-\delta + U(1)$. Alternatively one could use any continuation payoff after a reverse disagreement so long as it is smaller than or equal to $1 - \lambda$, which is the expected payoff from a coin flip without delay. The general analysis for any such game follows the same steps as in the present paper. In a sense, the critical part of incentive budget-breaking has to do with the costly delay that arises after a regular disagreement in which the uninformed types vote their ex ante favorite alternative in hope of persuading each other to switch, rather than the costly delay that happens after a reverse disagreement resulting from each tentatively agreeing with the other side.

Our result in Proposition 2 that re-voting yields welfare gains for moderate degrees of conflict, is robust to the maximum number of re-voting rounds allowed. Imagine that in the case of disagreement re-voting is allowed up to some fixed $T \geq 2$ rounds, after which a coin toss is used to decide between the two alternatives without further delay. Then, for beliefs of the uninformed γ just above 1/2, there is an interval of values of λ/δ such that the ex ante expected equilibrium payoff for the players is greater than the benchmark payoff of $1 - \lambda$ in the one-round voting game. The proof of this claim is similar to the analysis of the random switching equilibrium in the re-voting game in section 3.

An improvement in information aggregation is the source of welfare gains relative to the benchmark of one round voting, for both the re-voting game of section 3 and the repeated voting game of section 4. Such improvement is impossible, if either a decision has to be made immediately without delay, or if a decision has to be made within a fixed number of rounds and the delay cost is arbitrarily small. In particular, in the single-round voting game in section 2, the equilibrium outcome is a coin toss when the degree of conflict γ is greater than 1/2. In the re-voting game analyzed in section 3, Proposition 2 establishes that for any fixed γ greater than 1/2, welfare gains disappear when the delay cost is too small. Similarly, in the above-mentioned extension of the re-voting model, where re-voting is allowed for up to T rounds, when the delay cost δ between two rounds of voting converges to 0, the only equilibrium outcome converges to T rounds of regular disagreements followed by a coin toss in the last round. One interpretation is that costless straw polls or other forms of cheap talk cannot bring about any improvement in information aggregation or welfare, which is simply another illustration that in the environment of the present model information aggregation is impossible in any incentive compatible outcome without costly delay.

In contrast to Proposition 2 for the re-voting game, Proposition 6 establishes welfare gains with arbitrarily small delay cost in the repeated voting game. The latter result hinges on the assumption that the two players cannot commit to making a decision within finite number of rounds. Even though in equilibrium the expected duration of disagreement is finite, and in fact it takes a finite number of rounds of regular disagreement for the uninformed types to concede, the assumption of voting until agreeing creates a strictly positive payoff loss from delay as the delay cost goes to 0. This payoff loss reflects the role of incentive budget-breaking played by costly delay, even as the delay cost goes to 0.

We have shown that costly delay facilitates information aggregation and can yield welfare gains both with and without the commitment to making a decision after finitely many disagreements. This result naturally raises the question of when are the welfare gains maximized. We study this issue in a follow-up paper (Damiano, Li and Suen 2009) for a more general class of repeated proposal games using a continuous time framework.

The constructive role of costly delay is illustrated in the present paper with a model most suitable for small committees and bilateral negotiations. In an on-going project, we extend the re-voting model of section 3 to information aggregation in elections with two candidates, allowing any number of privately informed voters, and consider election rules that require a super majority to elect a candidate in the first round and a simple majority in the second and final round after a costly delay. In this extension, there are two states of the world, corresponding to which of the two candidates is the "right" one for all voters, and voters receive binary signals that are independent conditional on the state. We specify preferences and information structures of the voters in such a way that a voter that receives a signal supporting his preference bias votes according to his bias in each round, while a voter that receives the opposite signal chooses his vote by considering pivotal events. In addition to the standard kind of pivotal events in which the vote changes the outcome in the first round of voting (see, e.g., Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996), there is another kind of pivotal events: the vote causes delay and re-voting without affecting the eventual outcome, or avoids delay. As in the present paper, costly delay and re-voting can have a positive impact on the voters' ex ante welfare by improving information aggregation. Indeed, through careful choices of the super majority rule in the first round and the delay cost, we can arbitrarily closely approximate efficient information aggregation in an environment where little information aggregation is possible without costly delay.

Appendix

Proof of Proposition 1

We first verify the equilibrium conditions for the uninformed, in four cases.
(i) Consider first the case in which $x^1(\gamma) = 0$ and $\gamma' = 0$. For this to be an equilibrium, an uninformed type must prefer conceding to persisting. Since $V^0(1) = 1 - \lambda$ and $V^0(0) = 1$, the condition $U_p^1(\gamma, 0) \leq U_c^1(\gamma, 0)$ is equivalent to $\gamma \in [0, \delta/(2\delta + \lambda)]$.
(ii) Next, consider the case in which upon a regular disagreement the updated belief γ' belongs to $(0, 1/2)$. In this case, $V^0(\gamma') = 1 - \lambda\gamma'$, so there is a unique $\hat{x}^1(\gamma)$ such that $U_p^1(\gamma, \hat{x}^1(\gamma)) = U_c^1(\gamma, \hat{x}^1(\gamma))$, given by

$$\hat{x}^1(\gamma) = \frac{\gamma(\delta + \lambda) - (1 - \gamma)\delta}{2\gamma\delta}.$$

When $\gamma \geq \delta/(2\delta+\lambda)$, we have $\hat{x}^1(\gamma) \geq 0$. When $\gamma < 3\delta/(4\delta+\lambda)$, we have $\hat{x}^1(\gamma) < (1-\gamma)/\gamma$, so the resulting posterior γ' is indeed less than than $1/2$. Finally, $\hat{x}^1(\gamma) \geq 1$ is equivalent to $U_p^1(\gamma, 1) \geq U_c^1(\gamma, 1)$, so $x^1(\gamma) = 1$ is an equilibrium.
(iii) In the third case, $x^1(\gamma) = (1 - \gamma)/\gamma$ so that $\gamma' = 1/2$. When $\gamma \in [3\delta/(4\delta + \lambda), (3\delta + \lambda)/(4\delta + 2\lambda))$, there exists a continuation value $V^0(1/2) \in (1 - \lambda, 1 - \lambda/2]$ such that the indifference condition $U_p^1(\gamma, (1 - \gamma)/\gamma) = U_c^1(\gamma, (1 - \gamma)/\gamma)$ is satisfied. Finally, we need $(1 - \gamma)/\gamma \leq 1$, which implies that this case is valid only if $\gamma \geq \max\{1/2, 3\delta/(4\delta + \lambda)\}$.
(iv) When the updated belief γ' upon a regular disagreement is in $(1/2, 1]$, we have $V^0(\gamma') = 1 - \lambda$. There is a unique $\hat{x}^1(\gamma)$ such that $U_p^1(\gamma, \hat{x}^1(\gamma)) = U_c^1(\gamma, \hat{x}^1(\gamma))$, given by:

$$\hat{x}^1(\gamma) = \frac{\gamma(\delta + \lambda) - (1 - \gamma)(\delta + \lambda)}{2\gamma\delta}.$$

For $\gamma > (3\delta + \lambda)/(4\delta + 2\lambda)$, we have $\hat{x}^1(\gamma) \geq (1 - \gamma)/\gamma$, so that γ' indeed exceeds $1/2$. Moreover $\hat{x}^1(\gamma) \geq 1$ is equivalent to $U_p^1(\gamma, 1) \geq U_c^1(\gamma, 1)$, so $x^1(\gamma) = 1$ is an equilibrium.

Now we verify that the informed type has no incentive to deviate. When the uninformed type persists with probability x^1, the informed type's payoff from persisting is:

$$V_p^1(\gamma, x^1) = x^1(-\delta + W^0(\gamma')) + (1 - x^1),$$

and his payoff from conceding is:

$$V_c^1(\gamma, x^1) = x^1(1 - 2\lambda) + (1 - x^1)(-\delta + V^0(1)).$$

If $x^1 = 0$, the informed type clearly prefers persisting, with a payoff of 1, to voting conceding, with a payoff of $-\delta + V^0(\gamma)$. If $x^1 > 0$, we have $U_p^1(\gamma, x^1) \geq U_c^1(\gamma, x^1)$, or

$$\gamma[x^1(-\delta + U^0(\gamma') - 1 + 2\lambda) + (1 - x^1)(1 + \delta - U^0(1))] + (1 - \gamma)(-\delta + U^0(\gamma') - 1) \geq 0.$$

The second term of the above expression is strictly negative. Since $U^0(\gamma') \leq V^0(\gamma')$ for all γ' and $U^0(1) = V^0(1)$, the above inequality implies that

$$x^1(-\delta + V^0(\gamma') - 1 + 2\lambda) + (1 - x^1)(1 + \delta - V^0(1)) > 0,$$

which is equivalent to $V_p^1(\gamma, x^1) > V_c^1(\gamma, x^1)$.

Proof of Proposition 2

Consider first the case of a no switching equilibrium. An uninformed type is indifferent between persisting and conceding in the first round. Using the payoff from persisting, we have the equilibrium payoff of

$$U^1(\gamma) = \gamma(x^1(\gamma)(-\delta + 1 - \lambda) + (1 - x^1(\gamma))) + (1 - \gamma)(-\delta + 1 - \lambda).$$

An informed type receives a payoff of

$$V^1(\gamma) = x^1(\gamma)(-\delta + 1 - \lambda) + (1 - x^1(\gamma)).$$

For $\gamma > 1/2$, the payoff difference between the ex ante equilibrium payoff $W^1(\gamma)$ and the benchmark payoff of $W^0(\gamma) = 1 - \lambda$ is:

$$W^1(\gamma) - W^0(\gamma) = -\frac{1 - \gamma}{2 - \gamma}\delta + \frac{1}{2 - \gamma}(-x^1(\gamma)\delta + (1 - x^1(\gamma))\lambda).$$

After substituting the equilibrium value of $x^1(\gamma)$ given by in Proposition 1 into this expression, we obtain:

$$W^1(\gamma) - W^0(\gamma) = \frac{\delta}{2\gamma(2 - \gamma)}\left(-(2\gamma - 1)\rho^2 + 2(1 - \gamma)\rho + 1 - 4\gamma + 2\gamma^2\right),$$

where $\rho \equiv \lambda/\delta$. This is a quadratic expression in ρ, and it is non-negative if:

$$\rho \in \left[\frac{1-\gamma-\sqrt{\gamma(4\gamma^2-9\gamma+4)}}{2\gamma-1}, \frac{1-\gamma+\sqrt{\gamma(4\gamma^2-9\gamma+4)}}{2\gamma-1}\right].$$

The intersection between this region and the region $\gamma \in [(3+\rho)/(4+2\rho), 1]$ that supports a no switching equilibrium is

$$\rho \in \left(\frac{3-4\gamma}{2\gamma-1}, \frac{1-\gamma+\sqrt{\gamma(4\gamma^2-9\gamma+4)}}{2\gamma-1}\right]; \quad \gamma \leq 2-\sqrt{2}.$$

Next, consider the case of a random switching equilibrium. The equilibrium ex ante payoff of the players is

$$\begin{aligned} W^1(\gamma) =& \frac{\gamma}{2-\gamma}(x^1(\gamma)(-\delta+U^0(1/2))+(1-x^1(\gamma))) \\ &+ \frac{1-\gamma}{2-\gamma}(-\delta+U^0(1/2))+\frac{1-\gamma}{2-\gamma}(x^1(\gamma)(-\delta+V^0(1/2))+(1-x^1(\gamma))). \end{aligned}$$

Use $x^1(\gamma)=(1-\gamma)/\gamma$ and the equilibrium value of $U^0(1/2)$ that solves the indifference condition for a random switching equilibrium (with the corresponding value of $V^0(1/2)$), we obtain:

$$W^1(\gamma)-W^0(\gamma) = \frac{\delta}{\gamma(2-\gamma)}(\gamma^2-4\gamma+2).$$

So, $W^1(\gamma) \geq W^0(\gamma)$ if and only if $\gamma \leq 2-\sqrt{2}$. The region of welfare improvement that satisfies the condition $\gamma \in [3/(4+\rho), (3+\rho)/(4+2\rho))$ for a random switching equilibrium is therefore

$$\rho \in \left[\frac{3-4\gamma}{\gamma}, \frac{3-4\gamma}{2\gamma-1}\right]; \quad \gamma \leq 2-\sqrt{2}.$$

In a switching equilibrium, we have

$$\begin{aligned} W^1(\gamma)-W^0(\gamma) =& \frac{\gamma}{2-\gamma}(-x^1(\gamma)\delta+(1-x^1(\gamma))\lambda) \\ &+ \frac{1-\gamma}{2-\gamma}(\lambda-\delta)+\frac{1-\gamma}{2-\gamma}(x^1(\gamma)(-\delta+\lambda)+(1-x^1(\gamma))\lambda). \end{aligned}$$

Use the equilibrium value of $x^1(\gamma)$ given in Proposition 1, we obtain:

$$W^1(\gamma)-W^0(\gamma) = \frac{\delta}{2\gamma(2-\gamma)}\left(-(\rho\gamma-2(1-\gamma))^2+6\gamma^2-12\gamma+5\right)$$

This is a quadratic expression in ρ, and is non-negative when

$$\rho \in \left[\frac{1}{\gamma}\left(2(1-\gamma)-\sqrt{6\gamma^2-12\gamma+5}\right),\ \frac{1}{\gamma}\left(2(1-\gamma)+\sqrt{6\gamma^2-12\gamma+5}\right)\right].$$

The intersection between this region and the region that is consistent with the condition $\gamma \in (1/2, 3/(4+\rho))$ for a switching equilibrium is:

$$\rho \in \left[\frac{1}{\gamma}\left(2(1-\gamma)-\sqrt{6\gamma^2-12\gamma+5}\right),\ \frac{1}{\gamma}(3-4\gamma)\right); \quad \gamma \leq 2-\sqrt{2}.$$

Proof of Lemma 1

(i) For $k = 0$, we have $a_0 = 1$ and $b_0 = \delta + \lambda + \sqrt{\delta^2+\lambda^2} > 2\lambda$. Next, if $a_{k-1} \leq 1$ and $b_{k-1} > 2\lambda$, the two fractions that appear in the difference equation (11) are both positive. Hence $a_k \leq 1$ and $b_k > 2\lambda$ by induction

(ii) For the monotonicity of b_k, we can subtract b_{k-1} from both sides of the second equation in (11) to get:

$$b_k - b_{k-1} = -\frac{1+\delta-a_{k-1}+b_{k-1}}{b_0+1+\delta-a_k+b_k-2\lambda}(b_{k-1}-2\lambda) < 0.$$

To establish the monotonicity of a_k, we use induction. First, it is easy to see that $a_1 < a_0 = 1$. Next, assume that $a_{k-1} < a_{k-2}$. We can write:

$$\begin{aligned} a_k - a_{k-1} &= \frac{(1+\delta-a_{k-2})(b_0-2\lambda)}{b_0+1+\delta-a_{k-2}+b_{k-2}-2\lambda} - \frac{(1+\delta-a_{k-1})(b_0-2\lambda)}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda} \\ &< \frac{(1+\delta-a_{k-2})(b_0-2\lambda)}{b_0+1+\delta-a_{k-2}+b_{k-2}-2\lambda} - \frac{(1+\delta-a_{k-1})(b_0-2\lambda)}{b_0+1+\delta-a_{k-1}+b_{k-2}-2\lambda} \\ &< \frac{(1+\delta-a_{k-2})(b_0-2\lambda)}{b_0+1+\delta-a_{k-2}+b_{k-1}-2\lambda} - \frac{(1+\delta-a_{k-2})(b_0-2\lambda)}{b_0+1+\delta-a_{k-2}+b_{k-2}-2\lambda} = 0, \end{aligned}$$

where the first inequality follows from $b_{k-1} < b_{k-2}$, and the second inequality follows from the induction hypothesis and the fact that the second term is decreasing in a_{k-1}.

(iii) Solving for the steady state version of the difference equation (11), we obtain the steady state values $a_\infty = 1 + \lambda - \sqrt{\delta^2+\lambda^2}$ and $b_\infty = 2\lambda$. By the monotonicity of a_k and b_k, these steady state values are also the limit values of the sequence $\{(a_k, b_k)\}$.

(iv) By definition, we have $G_1 \in (0,1)$. Since $a_{k-1} \leq 1$ and $b_{k-1} > 2\lambda$, an induction argument establishes that $G_k \in (0,1)$ for all $k \geq 1$. Next, subtracting G_k from both sides of (10), we obtain

$$G_{k+1} - G_k = \frac{1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)}(1 - G_k) > 0.$$

(v) Since G_k is an increasing and bounded sequence, it has a limit value. By part (iii) established above, the limit is 1.

Proof of Proposition 4

We first establish the continuity of $U(\gamma)$ for all $\gamma < 1$. For each $k \geq 0$, the function $U(\gamma)$ is trivially continuous at any $\gamma \in (G_k, G_{k+1})$. We show by induction that $U(\gamma)$ is continuous at each G_{k+1}, that is,

$$a_{k+1} - b_{k+1}G_{k+1} = a_k - b_k G_{k+1}.$$

For $k = 0$, we have

$$a_1 - a_0 = -\frac{\delta(b_0 - 2\lambda)}{b_0 + \delta + b_0 - 2\lambda}; \quad b_1 - b_0 = -\frac{(b_0 + \delta)(b_0 - 2\lambda)}{b_0 + \delta + b_0 - 2\lambda}.$$

Therefore,

$$\frac{a_1 - a_0}{b_1 - b_0} = \frac{\delta}{b_0 + \delta} = G_1.$$

Next, denote $w_k = 1 + \delta - a_k + b_k - 2\lambda$. We have

$$\begin{aligned} a_{k+1} - a_k &= \frac{b_0 - 2\lambda}{w_k w_{k-1}}((a_k - a_{k-1})(b_0 + b_{k-1} - 2\lambda) + (1 + \delta - a_{k-1})(b_k - b_{k-1})); \\ b_{k+1} - b_k &= \frac{b_0 - 2\lambda}{w_k w_{k-1}}((a_k - a_{k-1})(b_{k-1} - 2\lambda) + (b_0 + 1 + \delta - a_{k-1})(b_k - b_{k-1}). \end{aligned}$$

Therefore,

$$\frac{a_{k+1} - a_k}{b_{k+1} - b_k} = \frac{1 + \delta - a_{k-1} + G_k(b_0 + b_{k-1} - 2\lambda)}{b_0 + 1 + \delta - a_{k-1} + G_k(b_{k-1} - 2\lambda)} = G_{k+1},$$

where the first equality follows from the induction hypothesis and the second equality follows from the law of motion of the sequence $\{G_k\}$ (equation 10). To show that $U(\gamma)$ is continuous at $\gamma = 1$, we note that $\lim_{k \to \infty} G_k = 1$ and $a_\infty - b_\infty = U(1)$. The continuity and monotonicity of $x(\gamma)$ follows immediately.

Proof of Lemma 2

From the proof of Proposition 3, $V(\gamma) = 1$ for $\gamma \in [0, G_1]$. Let $c_0 = 1$ and $d_0 = 0$. We derive difference equations for c_k and d_k by induction. For any $\gamma \in (G_k, G_{k+1}]$, $k \geq 1$, we can write

$$V(\gamma) = x(\gamma)\left(-\delta + c_{k-1} + d_{k-1}\frac{1-\gamma}{\gamma}\frac{1}{x(\gamma)}\right) + 1 - x(\gamma).$$

Using the formula (9) for $x(\gamma)$, we can verify the functional form of V and obtain a pair of difference equations in (c_k, d_k):

$$c_k = 1 - \frac{b_0(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}; \quad d_k = d_{k-1} + \frac{(1+\delta-a_{k-1})(1+\delta-c_{k-1})}{b_0+1+\delta-a_{k-1}+b_{k-1}-2\lambda}.$$

It is straightforward to show by induction that $c_k \leq 1$ and $d_k \geq 0$ for all k. This implies that $\{d_k\}$ is an increasing sequence. Let $w_k = 1+\delta-a_k+b_k-2\lambda$; that $\{c_k\}$ is a decreasing sequence follows immediately by induction if we establish that w_k is decreasing in k. To prove the latter claim, combine equations (11) to obtain

$$w_k = \delta + \frac{(b_0-2\lambda)w_{k-1}}{b_0+w_{k-1}}. \tag{A.1}$$

The derivative of the right-hand-side with respect to w_{k-1} is positive. So $w_{k-1} < w_{k-2}$ implies $w_k < w_{k-1}$. Now,

$$w_1 - w_0 = \delta - \frac{(1+\delta-a_0+b_0)w_0}{b_0+w_0} = -\frac{b_0(b_0-2\lambda)}{2b_0+\delta-2\lambda} < 0.$$

An induction argument then establishes the claim.

The limit value of c_k as k goes to infinity can be verified by using the difference equation for c_k and the limit values of a_k and b_k given in Lemma 1. To verify that the limit value of $d_k(1-G_k)/G_k$, we multiply both sides of the difference equation for d_k by $(1-G_k)/G_k$, and then use Bayes' rule for G_k and the fact that $\lim_{k\to\infty} G_k = 1$.

Proof of Proposition 5

We first establish a lemma.

LEMMA A. *As δ decreases, for any k: (i) $(1+\delta-a_k+b_k-2\lambda)/b_0$ decreases; (ii) a_k increases; (iii) b_k decreases; (iv) $(1+\delta-a_k)/b_0$ decreases; and (v) $(1+\delta-a_k)/(b_0+1+\delta-a_k+b_k-2\lambda)$ decreases.*

Proof. (i) Let $v_k = 1 + \delta - a_k$, $w_k = v_k + b_k - 2\lambda$, and $u_k = b_0 + w_k$. Recall that $b_0 = 1 + \delta - U(1)$, and therefore $\mathrm{d}b_0/\mathrm{d}\delta = 1 + \delta/\sqrt{\delta^2 + \lambda^2}$. Also, from the proof of Lemma 2 we know that w_k is decreasing in k.

First, we show that w_k is increasing in δ for each k. Take derivative of equation (A.1) to get

$$\frac{\partial w_k}{\partial \delta} = 1 + \frac{w_{k-1}(w_{k-1} + 2\lambda)}{(b_0 + w_{k-1})^2}\frac{\mathrm{d}b_0}{\mathrm{d}\delta} > 0; \quad \frac{\partial w_k}{\partial w_{k-1}} = \frac{b_0(b_0 - 2\lambda)}{(b_0 + w_{k-1})^2} > 0.$$

Now, we have

$$\frac{\mathrm{d}w_k}{\mathrm{d}\delta} = \frac{\partial w_k}{\partial \delta} + \frac{\partial w_k}{\partial w_{k-1}}\frac{\mathrm{d}w_{k-1}}{\mathrm{d}\delta}.$$

An induction argument establishes that $\mathrm{d}w_k/\mathrm{d}\delta > 0$ if we can show that $\mathrm{d}w_0/\mathrm{d}\delta > 0$, which is true because $w_0 = \delta + b_0 - 2\lambda$ is increasing in δ.

To establish part (i) of the lemma, we write $f_k = w_k/(b_0 + w_k)$. Use equation (A.1) for w_k to write:

$$f_k = \frac{\delta + f_{k-1}(b_0 - 2\lambda)}{b_0 + \delta + f_{k-1}(b_0 - 2\lambda)}.$$

The partial derivative $\partial f_k/\partial\delta$ has the same sign as

$$b_0 + (2f_{k-1}\lambda - \delta)\frac{\mathrm{d}b_0}{\mathrm{d}\delta}.$$

Since w_k is decreasing in k, we have that f_k is decreasing in k. Therefore, this expression is greater than

$$b_0 + (2f_\infty\lambda - \delta)\frac{\mathrm{d}b_0}{\mathrm{d}\delta},$$

which is positive, where $f_\infty = 1/2(1 - \lambda/(1 - \lambda + \delta - U(1)))$ is the limit value of f_k as k goes to infinity. It is also easy to see that f_k is increasing in f_{k-1}. The claim then follows if we show $\mathrm{d}f_0/\mathrm{d}\delta > 0$, which we can verify by using the definition of f_0 and taking derivatives with respect to δ.

(ii) We claim that $(b_0 - 2\lambda)/u_k$ is increasing in δ for each k. To prove it, let $t_k = w_k + 2\lambda$. Write the difference equation for w_k in the form:

$$\frac{t_k}{b_0 - 2\lambda + t_k} = \frac{(\delta + 2\lambda)u_{k-1} + (b_0 - 2\lambda)(t_{k-1} - 2\lambda)}{(b_0 + \delta)u_{k-1} + (b_0 - 2\lambda)(t_{k-1} - 2\lambda)}.$$

Let $g_k = (b_0 - 2\lambda)/u_k = 1 - t_k/u_k$. Then the above equation can be transformed into:

$$g_k = \frac{b_0 - 2\lambda}{\delta + 2b_0 - 2\lambda - b_0 g_{k-1}}.$$

It is clear that $\partial g_k/\partial g_{k-1} > 0$. Moreover, $\partial g_k/\partial \delta$ has the same sign as:

$$-(b_0 - 2\lambda) + (\delta + 2\lambda - 2\lambda g_{k-1})\frac{\mathrm{d}b_0}{\mathrm{d}\delta}.$$

Since g_k is increasing in k, the above expression is greater than

$$-(b_0 - 2\lambda) + (\delta + 2\lambda - 2\lambda g_\infty)\frac{\mathrm{d}b_0}{\mathrm{d}\delta} > 0,$$

where $g_\infty = 1/2(1-\lambda/(1-\lambda+\delta-U(1)))$ is the limit value of g_k as k goes to infinity. So an induction argument will establish the monotonicity of g_k with respect to δ if we establish that $\mathrm{d}g_0/\mathrm{d}\delta > 0$, which we can verify by using the definition of g_0 and taking derivatives with respect to δ.

To establish part (ii) of the lemma, we write the difference equation for a_k as:

$$a_k = 1 - g_{k-1}(1 + \delta - a_{k-1}).$$

Thus,

$$\frac{\mathrm{d}a_k}{\mathrm{d}\delta} = -g_{k-1} - (1 + \delta - a_{k-1})\frac{\mathrm{d}g_{k-1}}{\mathrm{d}\delta} + g_{k-1}\frac{\mathrm{d}a_{k-1}}{\mathrm{d}\delta}.$$

Since $\mathrm{d}a_0/\mathrm{d}\delta = 0$, an induction argument establishes that $\mathrm{d}a_k/\mathrm{d}\delta \leq 0$ for each k.

(iii) We write the difference equation for b_k as:

$$b_k = 2\lambda + g_{k-1}(b_{k-1} - 2\lambda),$$

implying that

$$\frac{\mathrm{d}b_k}{\mathrm{d}\delta} = (b_{k-1} - 2\lambda)\frac{\mathrm{d}g_{k-1}}{\mathrm{d}\delta} + g_{k-1}\frac{\mathrm{d}b_{k-1}}{\mathrm{d}\delta}.$$

We have already shown that $\mathrm{d}g_{k-1}/\mathrm{d}\delta > 0$. Moreover, $\mathrm{d}b_0/\mathrm{d}\delta > 0$. So an induction argument shows that $\mathrm{d}b_k/\mathrm{d}\delta > 0$ for each k.

(iv) From part (ii) we have v_k is increasing in δ for each k. Write the difference equation for a_k as:

$$\frac{v_k}{b_0} = \frac{\delta}{b_0} + g_{k-1}\frac{v_{k-1}}{b_0}.$$

Note that $v_0/b_0 = \delta/b_0$ is increasing in δ. Also, g_{k-1} is increasing in δ. So an induction argument establishes the claim.

(v) First, we claim that $v_k/(b_0-\lambda)$ is increasing in δ for each k. To prove it, write the difference equation for a_k as:

$$\frac{v_k}{b_0-\lambda} = \frac{\delta}{b_0-\lambda} + g_{k-1}\frac{v_{k-1}}{b_0-\lambda}.$$

Note that $v_0/(b_0-\lambda) = \delta/(b_0-\lambda)$ is increasing in δ. So an induction argument establishes the claim.

Next, we show that $(b_k-\lambda)/(b_0-\lambda)$ is decreasing in δ for each k. We can write the difference equation for b_k as:

$$\frac{b_k-\lambda}{b_0-\lambda} = \frac{\lambda}{b_0-\lambda} + \frac{(b_0-2\lambda)((b_{k-1}-\lambda)-\lambda)}{(b_0-\lambda)+(1+\delta-a_{k-1})+(b_{k-1}-\lambda)}.$$

Define $p_k = (b_k-\lambda)/(b_0-\lambda)$ and $q_k = (1+\delta-a_k)/(b_0-\lambda)$. Then we can write

$$p_k = (1-g_{k-1}) + \frac{b_0-2\lambda}{b_0-\lambda}\frac{p_{k-1}}{1+q_{k-1}+p_{k-1}}.$$

Note that

$$\begin{aligned}
\frac{\partial p_k}{\partial p_{k-1}} &= \frac{b_0-2\lambda}{b_0-\lambda}\frac{1+q_{k-1}}{(1+q_{k-1}+p_{k-1})^2} > 0,\\
\frac{\partial p_k}{\partial q_{k-1}} &= -\frac{b_0-2\lambda}{b_0-\lambda}\frac{p_{k-1}}{(1+q_{k-1}+p_{k-1})^2} < 0,\\
\frac{\partial p_k}{\partial g_{k-1}} &= -\frac{\lambda}{b_0-\lambda} < 0,\\
\frac{\partial p_k}{\partial b_0} &= -\frac{\lambda}{(b_0-\lambda)^2}\frac{1+\delta-a_{k-1}+\lambda}{u_{k-1}} < 0.
\end{aligned}$$

Now,

$$\frac{\mathrm{d}p_k}{\mathrm{d}\delta} = \frac{\partial p_k}{\partial b_0}\frac{\mathrm{d}b_0}{\mathrm{d}\delta} + \frac{\partial p_k}{\partial g_{k-1}}\frac{\mathrm{d}g_{k-1}}{\mathrm{d}\delta} + \frac{\partial p_k}{\partial q_{k-1}}\frac{\mathrm{d}q_{k-1}}{\mathrm{d}\delta} + \frac{\partial p_k}{\partial p_{k-1}}\frac{\mathrm{d}p_{k-1}}{\mathrm{d}\delta}.$$

Since $\mathrm{d}b_0/\mathrm{d}\delta > 0$, $\mathrm{d}g_{k-1}/\mathrm{d}\delta > 0$, $\mathrm{d}q_{k-1}/\mathrm{d}\delta > 0$ and $\mathrm{d}p_0/\mathrm{d}\delta = 0$, an induction argument establishes that $\mathrm{d}p_k/\mathrm{d}\delta < 0$ for each k.

To establish the last part of the lemma, we divide both the denominator and numerator of v_k/u_k by $b_0-\lambda$ to get:

$$\frac{v_k}{u_k} = \frac{q_k}{1+q_k+p_k}.$$

Since q_k is increasing in δ and p_k is decreasing in δ, the result follows. *Q.E.D.*

We are now ready to prove the proposition. Fix any $k \geq 1$. Let $w_k = 1+\delta-a_k+b_k-2\lambda$. For the effects on G_k, rewrite the difference equation for G_k as:

$$\frac{G_{k+1}}{1-G_{k+1}} = \frac{1+\delta-a_{k-1}}{b_0} + \frac{b_0+w_{k-1}}{b_0}\frac{G_k}{1-G_k}.$$

From part (i) and part (iv) of Lemma A, both w_k/b_0 and $(1+\delta-a_k)/b_0$ are increasing in δ. It is also clear that G_{k+1} is increasing in G_k. Finally, note that $G_1 = \delta/(b_0+\delta)$ is increasing in δ. An induction argument then establishes that G_k is strictly increasing in δ for each $k \geq 1$.

Next, for the effects on $x(\gamma)$, fix any γ and let

$$x_k(\gamma) = \frac{b_0}{b_0+w_{k-1}} - \frac{1-\gamma}{\gamma}\frac{1+\delta-a_{k-1}}{b_0+w_{k-1}}.$$

Since $x_k(G_{k+1}) = x_{k+1}(G_{k+1})$, and since

$$\frac{\partial x_k(\gamma)}{\partial\gamma} = \frac{1}{\gamma^2}\frac{1+\delta-a_{k-1}}{b_0+w_{k-1}} < \frac{1}{\gamma^2}\frac{1+\delta-a_k}{b_0+w_k} = \frac{\partial x_{k+1}(\gamma)}{\partial\gamma}$$

by part (v) of Lemma A, we obtain $x_k(\gamma) \geq x_{k+1}(\gamma)$ for all $\gamma \leq G_{k+1}$. Iterating the argument establishes that $x_k(\gamma) \geq x_{\tilde{k}}(\gamma)$ for all $\gamma \leq G_{k+1}$ and all $\tilde{k} \geq k$. The same argument also proves that $x_k(\gamma) \geq x_{\tilde{k}}(\gamma)$ for all $\gamma \geq G_k$ and all $\tilde{k} \leq k$. Combining these two results, we have $x_k(\gamma) \geq x_{\tilde{k}}(\gamma)$ for all $\tilde{k}$ if $\gamma \in (G_k, G_{k+1}]$. Now, for any $\tilde{\delta} > \delta$, denote the corresponding equilibrium strategy as $\tilde{x}(\gamma)$, and define $\tilde{x}_k(\gamma)$ analogously. Then, for any $\gamma \in (G_k, G_{k+1}]$,

$$x(\gamma) = x_k(\gamma) \geq x_{\tilde{k}}(\gamma) > \tilde{x}_{\tilde{k}}(\gamma) = \tilde{x}(\gamma),$$

where the first inequality follows because $\gamma \in (G_k, G_{k+1}]$, and the second inequality comes from part (i) and part (v) of Lemma A. Thus, $x(\gamma)$ is decreasing in δ for all γ.

Finally, for the effects on $U(\gamma)$, let $\tilde{d} > d$. Denote the sequence of threshold values of γ corresponding to $\tilde{d}$ as $\{\tilde{G}_k\}$, and denote the corresponding sequence of coefficients of the payoff function U as $\{(\tilde{a}_k, \tilde{b}_k)\}$. Suppose that $\gamma \in (G_k, G_{k+1}]$ while $\gamma \in (\tilde{G}_{\tilde{k}}, \tilde{G}_{\tilde{k}+1}]$. Then

$$\tilde{a}_{\tilde{k}} - \tilde{b}_{\tilde{k}}\gamma < a_{\tilde{k}} - b_{\tilde{k}}\gamma \leq a_k - b_k\gamma,$$

where the first inequality follows from part (ii) and part (iii) of Lemma A, and the second inequality follows from the convexity of $U(\gamma)$. Thus, $U(\gamma)$ is decreasing in δ.

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