POLARIZATION.

Polarization, or the tendency of economic agents to collect into different groups and sense increasing differences between those groups, is both cause and consequence of much economic behaviour. The essence of informal and formal club formation, polarization is born of an increasing sense of identity within the group members and an increasing sense of distance from members of other groups. The terminology is gaining increasing currency in economics, it has been employed in describing the diminution of the middle class in wage, income and wealth distributions, in studying growth and convergence issues, in examining the plight of the poor and it has been used in the study of inter-generational income relationships and in discriminating between competing matching models of marriage partners[1].

These literatures broadly interpret polarization as the disappearance of mass at the centre of an empirical distribution of a characteristic, or the increasing distance between and intensity of, multiple points of modality of the distribution as it evolves through time. It is inherently a dynamic process involving the comparison of the anatomy of states at different points in time, essentially comparing how the shape of the distribution of a characteristic (or a collection of characteristics) has evolved during the process. Thus the objective is to detect trends in shapes of distributions over time that reflect the polarization or de-polarization (sometimes referred to as convergence) of that group of agents. Though the polarization phenomenon is closely associated with trends in inequality it distinguishable from, and quite different to, changes in inequality in that increased polarization can induce either an increase, reduction or no change in inequality.

The concept need not be confined to the study of changes within a population's distribution of a particular characteristic, it can be used in assessing the relative movements of two or more distributions as they evolve (for example polarization between ethnic groups, genders, nations etc). In this context polarization takes the form of distributions becoming "less alike" in a particular fashion as such it involves comparison of complete distributions not just their location or scale characteristic. While the identification of polarization within and between populations presents quite distinct empirical challenges they have many common features which can be exploited in understanding the nature of the phenomenon. Indeed it is convenient to contemplate within distribution polarization phenomenon as the consequence of that population distribution being a mixture of sub-population distributions which are themselves polarizing (this notion is at the heart of the initial formalization of a Polarization index in Esteban and Ray (1994) and Duclos, Esteban and Ray (2004)). Such a construction will highlight why within distribution polarization is sometimes hard to detect in the absence of sub-population information.

Polarization: An Axiomatic Foundation

Indices of polarization were formulated in Esteban and Ray (1994) and Duclos, Esteban and Ray (2004) (see also Wang and Tsui (2000)) by positing a collection of axioms the consequences of

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1. Akerlov (1997), Anderson (2004) (2004a), Beach and Slotsve (1996), Beach, Chaykovske and Slotsve (1998), Bossert et. al. (2004), Corak (2004), D'Ambrosio and Wolff (2001), Foster and Wolfson (1992), Jenkins (1996), Jones (1997), Knack and Keefer (2001), Levy and Murnane (1992) Quah (1997) and Wolfson (1994), (1997) provide an extensive but not exhaustive list of usages of the notion of social distance and polarization.

which should be reflected in a Polarization measure. The axioms are founded upon what they termed an Identification-Alienation nexus wherein notions of polarization are fostered jointly by an agents sense of increasing within group identity and between group distance or alienation. The Axioms may be loosely summarized as follows:

1) A mean preserving reduction in the spread of a distribution cannot increase polarization.
2) Mean preserving reductions in the spread of sub-distributions at the extremes of a density cannot reduce polarization.
3) Separation of two sub densities toward the extremes of the distributions range must increase polarization.
4) Polarization measures should be population size invariant.

The polarization index developed for discrete distributions as a consequence of these axioms (Esteban and Ray (1994)) may be written as:

$$P_\alpha = K\sum_{i=1}^{n}\sum_{j=1}^{n} | x_i - x_j | \pi_i^{1+\alpha}\pi_j$$

Here K is a normalizing constant, π_i is the sample weight of the i'th observation and where $\alpha \geq 0$ is a polarization sensitivity factor chosen by the investigator. It may readily be seen that $\alpha = 0$ yields a sample weighted Gini coefficient.

The continuous distribution analogue (Duclos, Esteban and Ray (2004)) may be written as:

$$P_\alpha(F) = \int_y f(y)^\alpha \int_x | y - x | dF(x)dF(y)$$

Again α is the polarization sensitivity factor which in this case is confined to [0.25,1].

The Anatomy of Polarized States

To explore the anatomy of polarized states a population is represented by the equi-probable mixture of two sub distributions[2], representing two subgroups or clubs which make up the population. The initial sub distributions, which are identical except for having different means, are subjected to various transformations which are characterized in diagrams 1a through 4b.

Following the spirit of Axiom 3 the simplest form of polarization is when the sub-populations exhibit divergence in their means. Here there is no increase in within group identity but there is an increase in the distance between members of different groups (alienation). Diagram 1a exemplifies this situation in terms of the sub-populations and its consequence for the mixture is illustrated in diagram 1b. As may be seen the overlap of the two sub distributions diminishes and the centre of the mixture gets hollowed out. Note that when the means are relatively close together there is no hollowing out but simply a flattening of the uni-modal peak of the mixture distribution implying that polarized or polarizing states need not be characterized by the existence or emergence of bimodality (For example, for mixtures of equal variance normal

[2] Note that though the polarization phenomenon is not confined to two sub-groups, for simplicity of exposition the analysis will be in what follows.

distributions, bi-modality will not emerge under any mixing scheme until the difference in means exceeds $4.5^{0.5}$ standard deviations).Thus bump-hunting techniques available in the statistics literature (Good and Gaskins (1980), Hartigan and Hartigan (1985)) which seek out inflections in the probability density function will not necessarily be useful in the analysis of polarization.

Diagrams 2a and 2b illustrate another form of polarization when sub population means remain constant but their variances diminish. This is much in the spirit of Axiom 2 and characterizes a situation of increased identification within the groups without an increased sense of alienation between them. Again the overlap of the sub populations diminishes and the centre of the mixture gets hollowed out but in this case the anatomical change is not unequivocal. Finally when both locations and spreads remain constant but the lower distribution skews left and the upper distribution skews right the overlap again diminishes and the centre of the mixture distribution again hollows out as Diagrams 3a and 3b illustrate.

One thing these examples demonstrate is the potential for changes in the overlap measure to provide a general test or indicator of polarization, however this is not without qualification. Diagrams 4a and 4b return us to the increased identification case and demonstrate that, if the subgroup means happen to be close together, the extent of overlap can increase and the mass at the centre of the mixture distribution is enlarged with increased polarization. This highlights what is in effect a potential statistical identification problem associated with polarization when only the mixture of sub-distributions is observed. The tenuous link between polarization and inequality is also illustrated in this example. If we consider the mixture f(x) to be an equally weighted mixture of normal distributions $N(\mu_i,\sigma_i^2)$, i = 1,2, then the variance of x (for our purposes a measure of inequality) which will be $0.5(\sigma_1^2+\sigma_2^2+(\mu_1-\mu_2)^2)$ can be seen to either increase, decrease or remain unchanged with an increase in polarization interpreted as any combination in reductions of sub-population variances and increases in the difference of subpopulation means.

Alternative Between Group Polarization Measures.

Distributional Overlap.

The anatomy analysis suggests one technique for assessing polarization between two groups is to evaluate how much they have in common, such a measure corresponds to non – alienation and its negative (or some negative function of it) corresponds to a degree of alienation. Anderson (2004, 2004a) proposes an overlap measure as an index of convergence and a function of its negative as a measure of alienation. The extent to which two distributions f(x) and g(x) overlap is given by:

$$OV = \int_{-\infty}^{\infty} \min(f(x), g(x))dx$$

Clearly it is a number between 0 and 1 with 0 corresponding to no overlap and 1 to the perfect matching of the two distributions. It follows that 1-OV is a measure of the extent to which the distributions do not match or are alienated. When f(x) and f(y) are specified to the extent that all of their parameters can be estimated and the intersection points of f(x) and g(x) calculated OV can be estimated parametrically (see Anderson and Ge (2004)). When f( ) and g( ) are unknown, given independent samples from f( ) (represented by x) and g( ) (represented by y) of sizes n_x and

n_y respectively, its empirical counterpart may be implemented by choosing K + 1 mutually exclusive and exhaustive partitions of the range of x whose upper bound is defined by x^k, k = 1, …, K+1 and calculating:

$$OV^e = \sum_{i=1}^{k+1} \min\left(\frac{\sum_{j=1}^{n_x} I(x_j, x^i)}{n_x}, \frac{\sum_{j=1}^{n_y} I(y_j, x^i)}{n_y}\right)$$

Where $I(z,w^i)$ is an indicator function equal to 1 when z is in the interval (w^{i-1}, w^i) and 0 otherwise. The statistical properties of such an estimator are discussed in Anderson (2005). The non-parametric measure is prone to two sources of bias: the first, due to the intersection points of the underlying distributions not coinciding with partition points is actually not that large provided k is not small and the partition points are chosen judiciously, the second, due to the estimator being implicitly a conditional estimator can be large when the measure is either close to 0 or 1. However these biases do not appear to impede its use in calibrating changes in overlap. The main problem with this particular instrument arises when distributions do not actually overlap for that purpose the following measures may prove useful.

Gini Based Between Group Polarization Measures.

Starting with the classic Gini inequality coefficient which, with x_i being the income of the i'th agent for agents i = 1,..,n and where for convenience and without loss of generality, incomes are arranged in ascending rank order, may be written as:

$$Gini = \frac{1}{2n^2\mu}\sum_{i=1}^{n}\sum_{j=1}^{n} | x_i - x_j |$$

where μ is the mean of the x's. Suppose the rich and poor groups are defined by a poverty cut-off somewhere between x_p and x_{p+1} where 1 < p < n (what Yitzhaki (1994) refers to as perfect stratification of groups, i.e. no overlapping) then Gini may be thought of as the sum of the average mean normalized differences between agents in the poor group, between agents in the rich group and between poor and rich group agents. In measuring alienation it is only the last group of comparisons that are relevant, i.e. the average normalized difference between the rich group and poor group agents. In this case the new "PGini" index could be written as:

$$PGini = \frac{1}{p(n-p)\mu}\sum_{j=p+1}^{n}\sum_{i=1}^{p}(x_j - x_i)$$

Clearly this is still a number greater than 0 (but it is no longer guaranteed to be less than 1) which reflects the mean normalized average distance between the poor group and the rich group and as such it is easy to show that it is the overall mean normalized difference between the sub-group means Indeed the formulae can be generalized to general group differences where there is not perfect segmentation, i.e. where the subgroups overlap.

Observe that the same index would be arrived at if one were to work with $x_i - z$ and $x_j - z$, the corresponding distances from the poverty line z which facilitates a link to the well known family of poverty and welfare indices introduced by Foster, Greer and Thorbecke (1984) as follows. The formal representation of this family is given by:

$$POV_{\theta}(x,z)=\int_{0}^{z}\left(\frac{z-x}{z}\right)^{\theta}dF(x)$$

where F(x) is the cumulative density function (with p.d.f. f(x)) describing the population of incomes, z is the maximum of the poor and θ (≥ 0) is a parameter defining the nature of the poverty index and corresponds to a measure of poverty aversion. As a consequence POV_0 corresponds to the proportion of people in the poverty group, POV_1 is a normalized measure of the intensity of relative deprivation and so on. POV_i/POV_0 may be construed as the expected value of a weighted function of the normalized income deficiency where the weights are the (i-1)'th power of the normalized income deficiency itself. Thus increasing i increases the weights attached to those furthest from the poverty line interestingly as i becomes very large the index becomes a Rawlsian measure in focusing almost entirely on the poorest agent. All of these measures obey the Focus Axiom, that poverty measures should only depend upon the incomes of the poor. As such they are not in any sense related to the status of the rich.

Along similar lines RIC(x,z), an index of weighted relative distances of incomes above the poverty line may be contemplated whose theoretical representation is of the form:

$$RIC_{\theta}(x,z)=\int_{z}^{x\max}\left(\frac{x-z}{z}\right)^{\theta}dF(x)$$

In this case xmax corresponds to the maximum possible income. Here RIC_0 corresponds to the proportion of the population above the poverty line RIC_1 is a normalized measure of relative well being of the non-poor, RIC_2 is a measure of the intensity of the relative well being of those above the poverty line and so on. In this case as α becomes very large the index becomes almost entirely focused on the richest person, RIC_1/RIC_0 corresponds to the expected normalized income surplus over the maximum poverty income etc. For all θ > 0 all of these indices are essentially measuring relative weighted distances from the poverty line and it is in this sense that they are considered relative measures. However both RIC and POV are completely uninformed with respect to the distribution of incomes in the other group which accords with the focus axiom alluded to earlier. For the purposes of reflecting the notion of alienation between the poor and non-poor groups this axiom needs to be violated. Indeed the population analogue of PGini can be shown to be a specific member (θ = 1) of a general class of polarization measures defined by:

$$POL(z,\theta)=\left(\frac{z}{\mu}\right)^{\theta}\left(\frac{RIC_{\theta}}{RIC_{0}}-\frac{POV_{\theta}}{POV_{0}}\right)$$

Where θ ≥ 1.

Tests for Polarization.

Given the distribution of the above indices tests for increases or decreases in polarization in terms of movements in the indices can be readily established, however although indices provide complete orderings, much like the Gini coefficient with which they are associated they can be ambiguous. Direct tests of the anatomy of polarization based upon degrees of separation or stochastic dominance between density functions can provide an unambiguous (though not complete) orderings of the states of polarization. These tests can be developed by employing combinations of stochastic dominance conditions tests for which have been proposed by Anderson (1996) (2004), Davidson and Duclos (2000), Barrett and Donald (2003), Linton Maasoumi and Whang (2003), McFadden (1989). The conditions can be used in combination to compare the right separation of the upper distribution with the left separation of the lower distribution and thus establish a statistical criterion for polarization both within and between distributions. Anderson (2004a) provides a taxonomy of such tests.

An alternative Approach: the Growth and Convergence literature.

The endogenous growth literature has for a long while been concerned about issues of polarization specifically in the form of convergence or de-polarization. Early attempts at identifying the phenomenon via panel data regression techniques (see for example Barro (1998)) ran into difficulties in interpretation (Bernard and Durlauf (1996)). The phenomenon has been studied via the use of probability transition matrices implicit in the Markov chain methods employed by Quah (1997) (see also Durlauf and Quah (1999))[3]. Letting f(y) be the distribution of income y in some future period let f(x) be the distribution of income x in the present period the issue to be addressed is the relationship between the two distributions, i.e. the extent to, and manner in, which f(y) and f(x) are related. Thinking for the moment of x and y having the joint distribution f(y,x) so that f(y) and f(x) are the respective marginal distributions, at one extreme there is a sense of no relationship, that is to say when x and y are independent f(y,x) = f(y)f(x), at the other there is the completely deterministic environment whereby y = a + bx and the joint distribution is degenerate. Partitioning y and x into k mutually exclusive and exhaustive regions where p(y) and p(x) are respectively the vectors of marginal probabilities of falling into those regions, interest centers on the elements of the square matrix T defined by p(y) = T(y,x)p(x), the matrix in the square brackets in the following equation. T is of course the matrix of conditional probabilities formed by the product of the two square matrices in the equation so that:

$$\begin{pmatrix} p_1(y) \\ p_2(y) \\ . \\ p_k(y) \end{pmatrix} = \left[\begin{pmatrix} p_{11}(y,x) & p_{12}(y,x) & . & p_{1k}(y,x) \\ p_{21}(y,x) & p_{22}(y,x) & . & p_{2k}(y,x) \\ . & . & . & . \\ p_{k1}(y,x) & p_{k2}(y,x) & . & p_{kk}(y,x) \end{pmatrix} \begin{pmatrix} p_1(x) & 0 & & 0 \\ 0 & p_2(x) & & 0 \\ . & . & . & . \\ 0 & 0 & . & p_k(x) \end{pmatrix}^{-1} \right] \begin{pmatrix} p_1(x) \\ p_2(x) \\ . \\ p_k(x) \end{pmatrix}$$

Which is a matrix of conditional probabilities i.e. $T = \|p_{ij}(y,x)/p_j(x)\|$ i, j = 1, ..,k familiar in the convergence literature and made popular by Quah(1993). As such its properties are well known as are the techniques for its estimation. The i'th column of T is a conditional probability density function describing the distribution or re-allocation over states of the i'th element of p(x) the initial income distribution, to the elements of p(y), the resultant income distribution, after one period. If this process is thought to be time invariant then, letting p^s be the vector of $p_i(x)$'s s periods ahead then $p^s = T^s p$ corresponds to the s period ahead distribution and the solution to $p^\infty = Tp^\infty$ (if it exists) is what is known as the long run ergodic mass function. Interpreting these ergodic distributions as "characterizations of tendencies", one can infer a tendency towards polarization if they display multiple peaks. Polarization can be examined two ways in this

[3] These techniques have also been applied to the problem of intergenerational Income relationships (see Corak (2004)) and city sizes (Dobkins L. and Y. Ioannides, (2000), Anderson and Ge (2004)).

context. When the diagonal elements of T are large relative to the off diagonal elements the system is said to exhibit persistence, if the diagonal is particularly large in the high and low ends it indicates a tendency towards polarization. Alternatively one could compare p^{∞}, the long run distribution with with p(x), the initial distribution. If the former exhibits multiple peaks whereas the latter does not, a polarizing tendency may be inferred. One difficulty here is that no theory of inference has been outlined for examining the "multiple peakedness" of these ergodic functions as yet.

Multivariate Polarization

When agents are characterized in terms of more than one characteristic their polarization or otherwise will be reflected in more than one dimension. The empirical problem is then altogether much more challenging, the extension of the analysis to a multivariate measures can be somewhat problematic. Multivariate Gini coefficients have been developed (see Anderson(2004b) and Koshevoy and Mosler (1997)) but adapting them to the current context is complex, it requires defining a poverty cutoff for each characteristic (or some poverty boundary in multidimensional space), but even then extending the analogy to multivariate measures of FGT indices is not possible. One simple approach is to take a weighted geometric mean of the various AGini coefficients in each dimension but then the weights have to be determined in an inevitably arbitrary fashion.

On the other hand extension of the overlap measure OV to a multivariate overlap measure MOV is very straight forward since MOV is of the form:

$$MOV = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty} \min(f(x,y..,z),g(x,y..,z))dxdy..dz$$

In the corresponding empirical measure MOV^{e} , given suitable partitions in each dimension, the indicator function would simply be modified to a multivariate version accordingly (Anderson (2005) provides an example) and 1-MOV would provide an appropriate polarization measure.

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FIGURE 1. DIVERGENCE IN MEANS BETWEEN POPULATION POLARIZATION

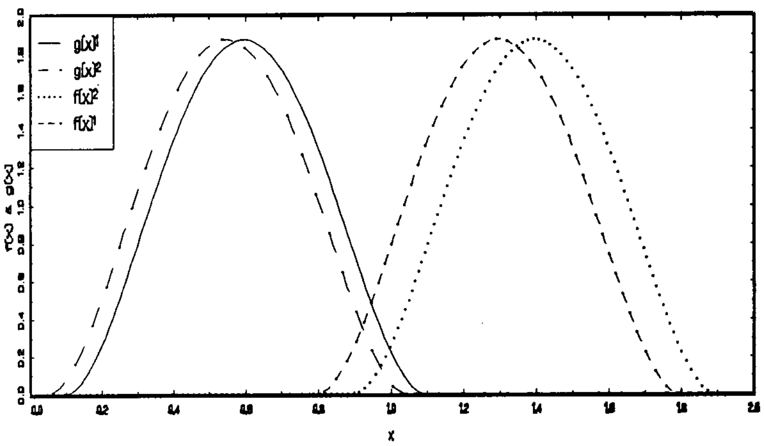


FIGURE 1a. DIVERGENCE IN MEANS WITHIN POPULATION POLARIZATION

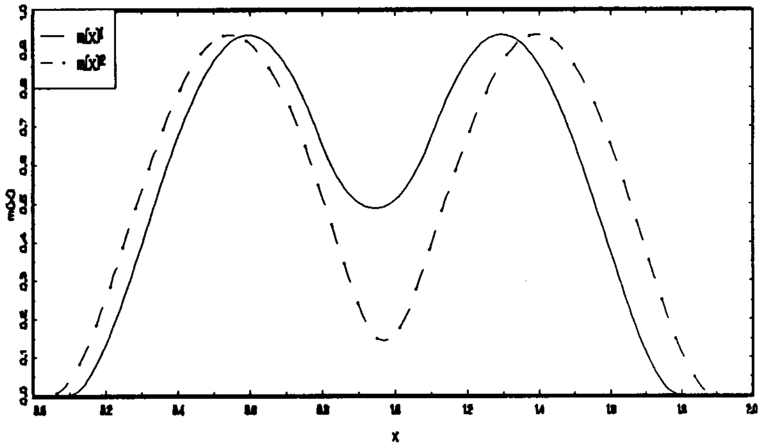


FIGURE 2. INCREASED CONCENTRATION BETWEEN POPULATION POLARIZATION

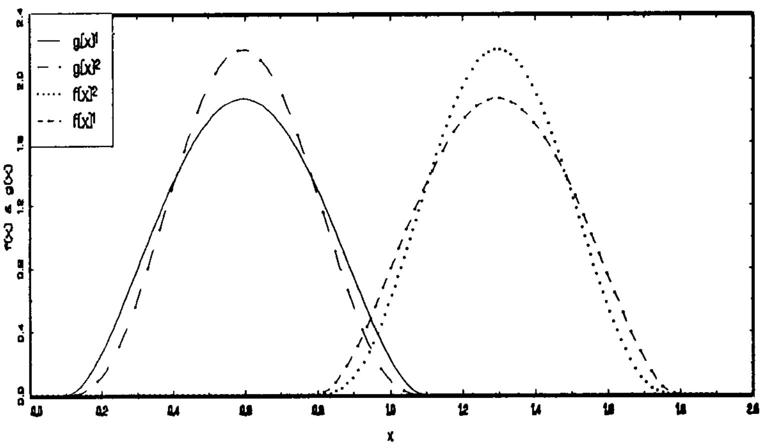


FIGURE 2a. INCREASED CONCENTRATION WITHIN POPULATION POLARIZATION

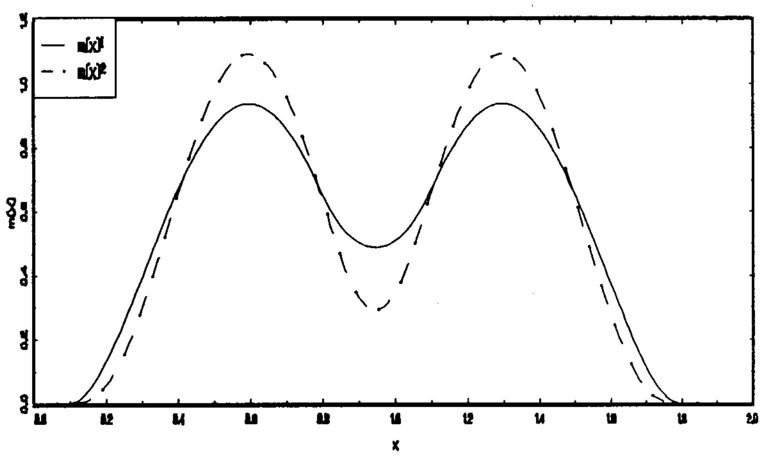


FIGURE 3. OPPOSITE SKEWNESS BETWEEN POPULATION POLARIZATION

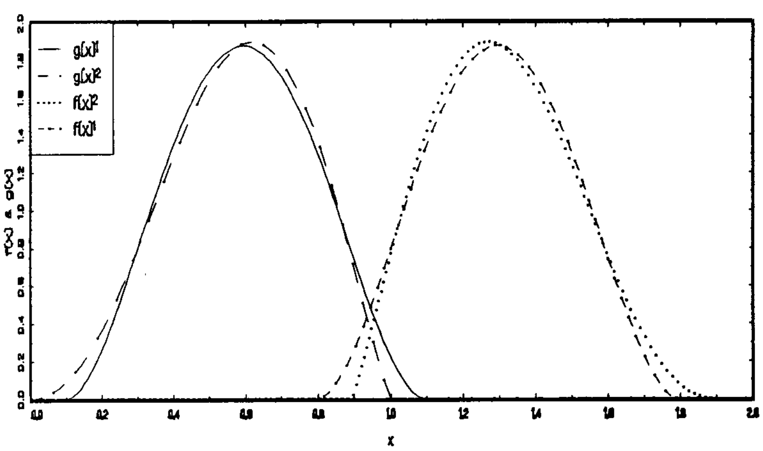


FIGURE 3a. OPPOSITE SKEWNESS WITHIN POPULATION POLARIZATION

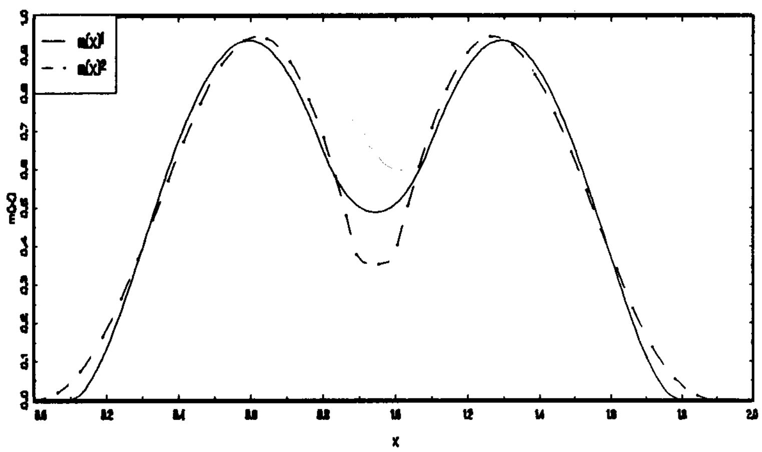
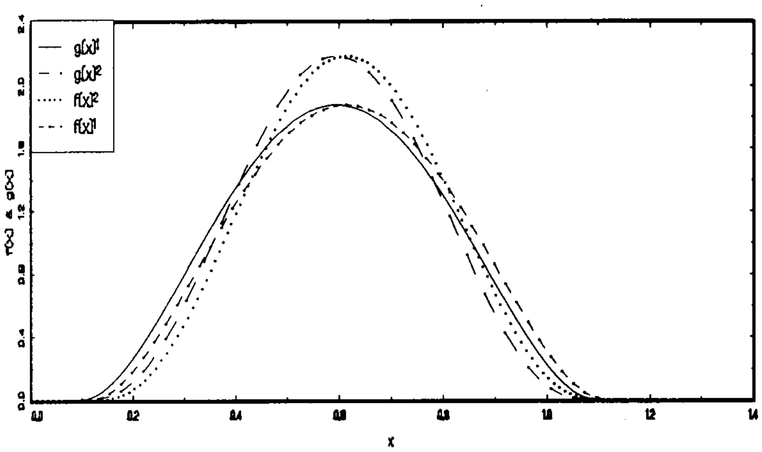


FIGURE 4. INCREASED CONCENTRATION BETWEEN POPULATION POLARIZATION CLOSE MEANS



4a. INCREASED CONCENTRATION WITHIN POPULATION POLARIZATION CLOSE MEANS